UNSOLVED CRUX PROBLEMS

As remarked in the problem section, no problem is ever closed. We always accept new solutions and generalizations to past problems. Chris Fisher published a list of unsolved problems from ***Crux*** *[2010 : 545, 547]. Below is a sample of two of these unsolved problems.*

283$\star$. [1977 : 250 ; 1978 : 115, 195–196]

Proposed by A. W. Goodman, University of South Florida.

The function

$$y = -\frac{2x \ln x}{1 - x^2}$$

is increasing for $0 < x < 1$ and, in fact, y runs from 0 to 1 in this interval. Therefore an inverse function $x = g(y)$ exists. Can this inverse function be expressed in closed form and if so what is it ? If it cannot be expressed in closed form, is there some nice series expression for $g(y)$? The series need not be a power series.

1581$\star$. [1990 : 266 ; 1991 : 308–309]

Proposed by Murray S. Klamkin and Andy Liu, University of Alberta.

If T_1 and T_2 are two triangles with equal circumradii, it is easy to show that if the angles of T_2 *majorize* the angles of T_1, then the area and perimeter of T_2 is not greater than the area and perimeter, respectively, of T_1. (One uses concavity of $\sin x$ and $\log \sin x$ in $(0, \pi)$.) If T_1 and T_2 are two tetrahedra with equal circumradii, and the solid angles of T_2 majorize the solid angles of T_1, is it true that the volume, the surface area, and the total edge length of T_2 are not larger than the corresponding quantities for T_1 ?

Editor's comment. For any n-set $\{a_1, a_2, \ldots, a_n\}$ let the elements be sorted into order as $a_{(1)} \leq a_{(2)} \leq \cdots \leq a_{(n)}$. A set $\{a_1, a_2, \cdots, a_n\}$ *majorizes* $\{b_1, b_2, \cdots, b_n\}$ if for all $m \in \{1, \ldots, n\}$ we have $\sum_{i=m}^{n} a(i) \geq \sum_{i=m}^{n} b(i)$.