BOOK REVIEWS

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Enlightening Symbols : A Short History of Mathematical Notation and Its Hidden Powers by Joseph Mazur
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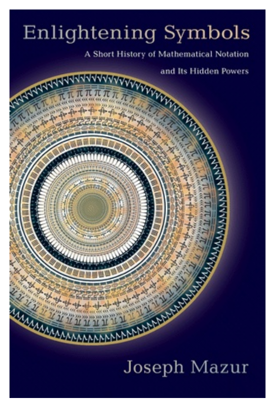
Enlightening Symbols is a captivating book on the evolution of the mathematical notations. Compared to classical works on the topics, such as F. Cajori's *History of mathematical notation*, this book is much more of an easy read. It focuses on just a few mathematical notations (the numeric systems, the basic operations, and simple equations) but offers a thorough treatment around them including the historical context in which the mathematical works were written and read, as well as some graphical extracts of the ancient works. This way, one has an idea how the symbols impacted the way of thinking at that time.



Imagine you could use only words to describe and solve polynomial equations in one variable... That is how the Pythagoreans and Euclid wrote and communicated. The book provides this example : "Given a sum of three quantities and also the sums of every pair containing one of those specified quantities, then that specified quantity is equal to the difference between the sums of those pairs and the total sum of the original three quantities". The author compares how almost any college student would be able to solve this using x, y, and z (and a, b, c the indicated sums) and how such a recipe, called *flower of Thymaridas*, was made available at the times of the Pythagoreans so as to solve it.

The book by J. Mazur starts with a fairly long exposition on the various number systems : babylonian, greek, roman, hebrew, aztec, chinese, indian... For a few of them, operations are shown, and this is where one really meets these numbers, being able to speak about their "capabilities" (e.g. how they can be manipulated to perform long additions, either graphically or on abacus). For a while, the hazardous ways through which the indo-arabic numeral system came through Europe are explored. This part is a bit hard to read as it has a few repetitions which circle around the question of original entry ; however, it sets the stage and provides some expressive descriptions of numbers, such as the hands of merchants of Western Europe negotiating with merchants from further east without understanding much of the rest of the language.

After the numbers, the symbols of early algebra are explored from word-based algebra (until our x and y, or a and b) with quite a number of imaginative ways presented through the eyes of Diophantus, R. Bombelli, C. Rudolff, or F. Viète. The link to geometry is omnipresent, and indeed this is how a square or a product was considered, but the expressivity of algebra is being developed, until the regularity of combining products of powers of the unknown (adding the exponents), until the resolution of equations becomes easiest, until... the fundamental theorem of algebra and the introduction of the square root of -1. G. Leibnitz and I. Newton conclude the panaché of notations, all leading to the notations currently in use in much of the western world. Then follows a part where less mathematical assertions are made, and more perception and psychology is explored. J. Mazur describes how the perception of symmetry and other patterns influences our perception of formulae, he describes an experiment by himself as well as several other psychologists or neuroscientists, concluding a potential evidence of a sense of reading mathematical formulae that may be transmitted through evolution.

Unfortunately, the book does not mention the differences of notations across the cultures and languages (such as the various ways of doing the long division, or the usage of j for the root of -1 in electrical engineering). However, this is a rather natural consequence of the book ; notations have evolved differently depending on the usages. Neither does the book offer sufficient material for proposing problems to students. However, most of the references are well documented, with URLs given when possible. This should allow a teacher to go and understand the ancient works so as to propose introductory and exercise materials. Certainly such a continuation of the book would be very interesting to share in a wiki space such as the census of mathematical notations (`http://wiki.math-bridge.org/display/ntns/`).

A tiny note is offered here to the readers who, similarly to me, like to read the electronic versions of the book. Reading on a small mobile device is an option as the eBook version is available (from Google Play Books as ePub and PDF and many others). However, some characters may be missing on the mobile version (such as the inverse psi to indicate a minus in Diophantus times) ; moreover, the ePub version suffers from missing characters in the formulae and words, with which normal mathematicians can easily cope but need some vigilance. The PDF does not have this issue but is non-searchable.

Overall, I recommend this book to anyone thinking about the mathematical notation they use and could use. It may even be a reading for senior college students. For those who think that we should abstain from discussing and varying mathematical notations, the book provides ample illustrations of how deeply notation influences our conceptualization ; the author concludes that “routine and familiarity are the tailwind of conceptions.”

Let me conclude with an example of the the author’s delightful style. He describes the introduction of the terminology for *complex numbers* with their *real* and *imaginary* parts, which is now standard ; he then editorializes that these names are unfortunate “because they are the names of classes of numbers that are neither imaginary nor complex.”