THE OLYMPIAD CORNER

No. 313

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The problems featured in this section have appeared in a regional or national mathematical Olympiad. Readers are invited to submit solutions, comments and generalizations to any problem. Electronic submissions are preferable, with each solution contained in a separate file. Files should be named using the convention LastName_FirstName_OCProblemNumber *(example Doe_Jane_OC1234.tex). It is preferred that readers submit a LaTeX file and a pdf file for each solution, although other formats, such as Microsoft Word, are also accepted. Readers are invited to email solutions and contests to the editor at* `crux-olympiad@cms.math.ca`*. Submissions by regular mail are also accepted and should be sent to the address inside the back cover. Name(s) of solver(s) with affiliation, city, and country should appear on each solution, and each solution should start on a separate page.*

To facilitate their consideration, solutions to the problems should be received by the editor by **1 September 2014***, although solutions received after this date will also be considered until the time when a solution is published.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, 7, and 9, English will precede French, and in issues 2, 4, 6, 8, and 10, French will precede English. In the solutions' section, the problem will be stated in the language of the primary featured solution.

The editor thanks Rolland Gaudet, of l'Université Saint-Boniface in Winnipeg, for translations of the problems.

OC131. Find all $f, g : \mathbb{R} \to \mathbb{R}$ such that

$$g(f(x) - y) = f(g(y)) + x,$$

for all $x, y \in \mathbb{R}$.

OC132. Find all primes p and q such that

$$(p+q)^p = (q-p)^{(2q-1)}.$$

OC133. Let $f(x) = (x+a)(x+b)$ where $a, b > 0$. Find the maximum of

$$F = \sum_{1 \leqslant i < j \leqslant n} \min \{f(x_i), f(x_j)\},$$

where $x_1, x_2, \ldots, x_n \geqslant 0$ are real numbers satisfying $x_1 + x_2 + \ldots + x_n = 1$.

OC134. Let ABC be an acute-angled triangle with $AB \neq AC$. Let Γ be the circumcircle of ABC, H the orthocentre of ABC and O the centre of Γ. Let M be the midpoint of BC. The line AM meets Γ again at N and the circle with diameter AM crosses Γ again at P. Prove that the lines AP, BC and OH are concurrent if and only if $AH = HN$.

OC135. Prove that for each $n \in \mathbb{N}$ there exist natural numbers $a_1 < a_2 < \cdots < a_n$ such that $\phi(a_1) > \phi(a_2) > \cdots > \phi(a_n)$ where ϕ denotes the Euler ϕ function.

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OC131. Déterminer toutes $f, g : \mathbb{R} \to \mathbb{R}$ telles que

$$g(f(x) - y) = f(g(y)) + x$$

pour tous $x, y \in \mathbb{R}$.

OC132. Déterminer tous les nombres premiers p et q tels que

$$(p+q)^p = (q-p)^{(2q-1)} .$$

OC133. Soit $f(x) = (x+a)(x+b)$ où $a, b > 0$. Déterminer le maximum de

$$F = \sum_{1 \leqslant i < j \leqslant n} \min \{f(x_i), f(x_j)\} ,$$

où $x_1, x_2, \ldots, x_n \geqslant 0$ sont des nombres réels satisfaisant $x_1 + x_2 + \ldots + x_n = 1$

OC134. Soit ABC un triangle à angles aigus tel que $AB \neq AC$. Soit Γ le cercle circonscrit de ABC, H l'orthocentre de ABC, O le centre de Γ, et M le mipoint de BC. La ligne AM rencontre Γ de nouveau à N. Le cercle avec diamètre AM croise Γ de nouveau à P. Démontrer que les lignes AP, BC et OH sont concourantes si et seulement si $AH = HN$.

OC135. Démontrer que pour tout $n \in \mathbb{N}$ il existe des nombres naturels $a_1 < a_2 < \cdots < a_n$ an tels que $\phi(a_1) > \phi(a_2) > \cdots > \phi(a_n)$ où ϕ dénote la fonction ϕ d'Euler.