THE CONTEST CORNER

No. 15

Shawn Godin

The problems featured in this section have appeared in, or have been inspired by, a mathematics contest question at either the high school or the undergraduate level. Readers are invited to submit solutions, comments and generalizations to any problem. Electronic submissions are preferable, with each solution contained in a separate file. Files should be named using the convention LastName_FirstName_CCProblemNumber *(example Doe_Jane_CC1234.tex). It is preferred that readers submit a LaTeX file and a pdf file for each solution, although other formats, such as Microsoft Word, are also accepted. Readers are invited to email solutions and contests to the editor at* `crux-contest@cms.math.ca`. *Submissions by regular mail are also accepted and should be sent to the address inside the back cover. Name(s) of solver(s) with affiliation, city, and country should appear on each solution, and each solution should start on a separate page.*

To facilitate their consideration, solutions to the problems should be received by the editor by **1 September 2014**, *although solutions received after this date will also be considered until the time when a solution is published.*

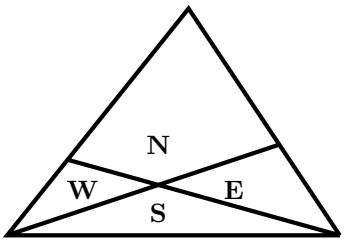
Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, 7, and 9, English will precede French, and in issues 2, 4, 6, 8, and 10, French will precede English. In the Solutions section, the problem will be stated in the language of the primary featured solution.

The editor thanks André Ladouceur, Ottawa, ON, for translations of the problems.

CC71. A bag is filled with red and blue balls. Before drawing a ball, there is a $\frac{1}{4}$ chance of drawing a blue ball. After drawing out a ball, there is now a $\frac{1}{5}$ chance of drawing a blue ball. How many red balls are in the bag?

CC72. From the set of natural numbers $1, 2, 3, \ldots, n$, four consecutive even numbers are removed. The remaining numbers have an average value of $51\frac{9}{16}$. Determine all sets of four consecutive even numbers whose removal creates this situation.

CC73. A farmer owns a triangular field, as shown. He reckons 5 sheep can graze in the west field, 10 sheep can graze in the south field, and 8 can graze in the east field. (All sheep eat the same amount of grass.) How many sheep can graze in the north field?



CC74. Let $1000 \leq n = ABCD_{10} \leq 9999$ be a positive integer whose digits $ABCD$ satisfy the divisibility condition:

$$1111 \mid (ABCD + AB \times CD).$$

Determine the smallest possible value of n.

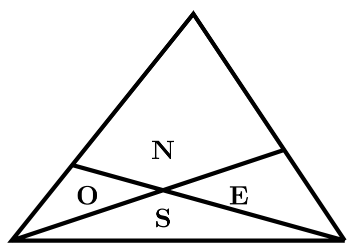
CC75. Let P be a point inside the triangle ABC such that $\angle PAC = 10°$, $\angle PCA = 20°$, $\angle PAB = 30°$ and $\angle ABC = 40°$. Determine $\angle BPC$.

. . . .

CC71. Un sac contient des boules rouges et des boules bleues. Si on pige au hasard une boule du sac, la probabilité de choisir une boule bleue est de $\frac{1}{4}$. Après avoir pigé une boule, la probabilité de choisir une boule bleue est maintenant de $\frac{1}{5}$. Combien y a-t-il de boules rouges dans le sac ?

CC72. On enlève quatre entiers pairs consécutifs de l'ensemble contenant les entiers positifs $1, 2, 3, \ldots, n$. Les nombres qui restent ont une moyenne de $51\frac{9}{16}$. Déterminer tous les ensembles de quatre entiers pairs consécutifs que l'on aurait pu enlever.

CC73. Un fermier possède un champ de forme triangulaire, comme dans la figure ci-dessous. Il calcule que 5 brebis peuvent brouter dans le champ ouest, 10 brebis peuvent brouter dans le champ sud et 8 brebis peuvent brouter dans le champ est. (Toutes les brebis mangent la même quantité d'herbe.) Combien de brebis peuvent brouter dans le champ nord ?



CC74. Soit n ($1000 \leq n = ABCD_{10} \leq 9999$) un entier positif dont les chiffres $ABCD$ vérifient la condition de divisibilité suivante :

$$1111 \mid (ABCD + AB \times CD).$$

Déterminer la plus petite valeur possible de n.

CC75. Soit P un point à l'intérieur du triangle ABC de manière que $\angle PAC = 10°$, $\angle PCA = 20°$, $\angle PAB = 30°$ et $\angle ABC = 40°$. Déterminer la mesure de l'angle BPC.