THE CONTEST CORNER

No. 5

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The Contest Corner *is a new feature of* ***Crux Mathematicorum****. It will be filling the gap left by the movement of* Mathematical Mayhem *and* Skoliad *to a new on-line journal in 2013. The column can be thought of as a hybrid of* Skoliad, The Olympiad Corner *and the old* Academy Corner *from several years back. The problems featured will be from high school and undergraduate mathematics contests with readers invited to submit solutions. Readers' solutions will begin to appear in the next volume.*

Solutions can be sent to:

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or by email to

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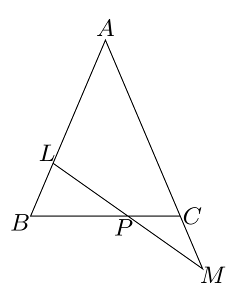
The solutions to the problems are due to the editor by **1 November 2013**.

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, 7, and 9, English will precede French, and in issues 2, 4, 6, 8, and 10, French will precede English. In the solutions' section, the problem will be stated in the language of the primary featured solution.

The editor thanks Rolland Gaudet of Université de Saint-Boniface, Winnipeg, MB for translating the problems from English into French.

CC21. In the diagram ΔABC is isosceles with $AB = AC$. Prove that if $LP = PM$, then $LB = CM$.



CC22. Points $A_1, A_2, \ldots, A_{2k}$ are equally spaced around the circumference of a circle and $k \geq 2$. Three of these points are selected at random and a triangle is formed using these points as its vertices. Determine the probability that the triangle is acute.

CC23. The three-term geometric progression $(2, 10, 50)$ is such that

$$(2 + 10 + 50) \times (2 - 10 + 50) = 2^2 + 10^2 + 50^2.$$

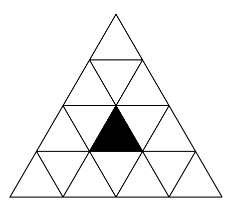
(a) Generalize this (with proof) to other three-term geometric progressions.
(b) Generalize (with proof) to geometric progressions of length n.

CC24. Given the equation

$$x^4 + y^4 + z^4 - 2y^2z^2 - 2z^2x^2 - 2x^2y^2 = 24.$$

(a) Prove that the equation has no integer solutions.
(b) Does this equation have rational solutions? If yes, give an example. If no, prove it.

CC25. Alphonse and Beryl are playing a game, starting with the geometric shape shown. Alphonse begins the game by cutting the original shape into two pieces along one of the lines. He then passes the piece containing the black triangle to Beryl, and discards the other piece. Beryl repeats these steps with the piece she receives; that is to say, she cuts along the length of a line, passes the piece containing the black triangle back to Alphonse, and discards the other piece. This process continues, with the winner being the player who, at the beginning of his or her turn, receives only the black triangle. Is there a strategy that Alphonse can use to be guaranteed that he will win?



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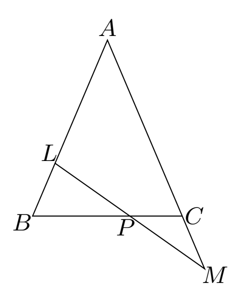
CC21. Dans le diagramme, ΔABC est isocèle tel que $AB = AC$. Démontrer que si $LP = PM$, alors $LB = CM$.



CC22. Les points $A_1, A_2, \ldots, A_{2k}$ sont distribués à distances égales sur la circonférence d'un cercle ; aussi, $k \geq 2$. Si trois de ces points sont choisis au hasard et si un triangle est formé avec ces points comme sommets, déterminer la probabilité que ce triangle est aigu.

CC23. Trois termes d'une progression géométrique $(2, 10, 50)$ sont tels que

$$(2 + 10 + 50) \times (2 - 10 + 50) = 2^2 + 10^2 + 50^2.$$

(a) Généraliser ce résultat à d'autres progressions géométriques dont on donne trois termes et en fournir une preuve.
(b) Généraliser ce résultat à d'autres progressions géométriques dont on donne n termes et en fournir une preuve.

CC24. Soit l'équation

$$x^4 + y^4 + z^4 - 2y^2z^2 - 2z^2x^2 - 2x^2y^2 = 24.$$

(a) Démontrer que cette équation n'a aucune solution entière.
(b) Cette équation a-t-elle solution(s) rationnelle(s) ? Si oui, en fournir une. Si non, démontrer qu'il n'y en a pas.

CC25. Alphonse et Bernard s'amusent à un jeu qui démarre avec la forme géométrique indiquée. Alphonse commence le jeu en taillant la forme en deux suivant une de ses lignes. Il donne alors à Bernard le morceau qui contient le triangle noir, mettant l'autre morceau à la poubelle. Bernard répète la taille suivant une ligne et remet le morceau avec le triangle noir à Alphonse, mettant l'autre à la poubelle. Le processus continue ainsi. Le joueur gagnant est celui qui reçoit un simple triangle noir au début de son jeu. Alphonse a-t-il une stratégie qui assure qu'il va gagner ?