

BOOK REVIEWS

John Grant McLoughlin

First Steps for Math Olympians: Using the American Mathematics Competitions

By J. Douglas Faires, Mathematical Association of America, 2006

ISBN 0-88385-824-X, hardcover, 307+xxii pages, US\$46.50

Reviewed by **Robert D. Poodiack**, Norwich University, Northfield, Vermont, USA

This excellent book adds to the problem-solving literature not only as a source of problems and solutions, but also as a primer of concepts and techniques for problem-solving. The book is limited, however, to pre-calculus mathematics.

In the introduction, Professor Faires writes about the history of, and his own involvement with, the American High School Mathematics Examination. This examination, now split into grade levels as part of the American Mathematics Competition (AMC), has been widely used as a springboard for students toward taking the American Invitational Mathematics Examination (AIME). Students who do well enough on the AIME may be asked to participate in the United States of America Mathematical Olympiad and the International Mathematical Olympiad.

The book is split into 18 chapters, each dealing with a single topic. The topics range through arithmetic and algebra, functions and geometry, sequences and series, probability and statistics, and trigonometry and basic number theory. Most chapters begin with an enumeration of basic definitions and results. Few formal proofs are included, most likely for brevity. For many results, though, an idea of why the result is true is included. As the target audience is high school students, this is not a large problem.

In each chapter, major theorems are presented for drill purposes. By organizing the book according to topics rather than presenting old AMC exams in chronological order, the author has made this book easier to use than the various MAA olympiad question books (where important results are listed in alphabetical order in an appendix). The sheer number of results can be mind-boggling, especially in the geometry sections, but they will all be recognized as useful by anyone who has studied mathematics competition problem collections. (I had only a vague recollection of the Tangent Chord Theorem or the External Secant Theorem myself!)

The meat of the book is, of course, in the problems. All the problems in the book are taken from old AMC exams. Each chapter ends with three completely solved "examples," in increasing order of difficulty, followed by a set of 10 related exercises, whose answers are in the back of the book. As in the actual AMC exams, all of the examples and exercises are multiple-choice. The problems and solutions presented are quite stimulating and ingenious.

Topics worthy of student research are sometimes introduced in a disguised manner.

For instance, Exercise 8 in Chapter 12, on sequences and series, reads:

Alice, Bob, and Carol repeatedly take turns tossing a fair regular six-sided die. Alice begins; Bob always follows Alice; Carol always follows Bob; and Alice always follows Carol. Find the probability that Carol will be the first to toss a six.

In addition to the solution involving an infinite geometric series, this example can lead to a discussion of probability “trees” that contain “wreaths” in their structure. If we change the probabilities of success for each person, we enter the theory of three-way duels!

In another example, in the chapter on functions, students are asked to find the maximum value of

$$f(x) = \sqrt{8x - x^2} - \sqrt{14x - x^2 - 48}.$$

The author presents a canny solution involving the geometry of semi-circles and, most notably, *no calculus*.

In his solution to the previous problem, Professor Faires makes a point of noting that “no problem on the AMC has a calculus solution that is easier than some non-calculus solution”. Given the intended audience of high-school and middle-school students, the AMC exams and this book leave out calculus altogether, as well as other advanced topics. University competitors will want to augment this book with another more advanced competition book.

My only complaints about the book are organizational. Since the full solutions to all of the exercises are given in the back of the book, it would have been nice for the questions and answer choices to be reprinted with them. For old forgetful folks like me, some cross-referencing of page numbers between questions and answers would have been welcome.

These are mere quibbles, though. *First Steps for Math Olympians* is an excellent introductory primer for any precocious student gearing up for the AMC exam. Professor Faires has provided a well-organized, easily digested study guide appropriate for high-school and middle-school students as well as any college student just starting out in competitions. The volume and quality of problems make the book worthwhile for students and coaches alike. I plan on paying the ultimate compliment of filching some of these problems for a competition I organize this year!

The Edge of the Mathematical Universe:

Celebrating 10 Years of Math Horizons

Edited by Deanna Haunsperger and Stephen Kennedy, Mathematical Association of America, 2006

ISBN 0-88385-550-0, hardcover, xii+303 pages, US\$57.50.

Reviewed by **John Grant McLoughlin**, University of New Brunswick, Fredericton, NB.

Wow! Some books arrive that have me saying, "I'll review that one!" This is one of those books. The collection of articles and the overall presentation of *The Edge of the Mathematical Universe* invite distraction from other activities as one engages with the mathematical playfulness evident on its pages. The book is a tribute to the MAA publication, *Math Horizons*. The editors, Deanna Haunsperger and Stephen Kennedy, guided *Math Horizons* through the second half of its first decade when they took over from founding editor Donald Albers. Here they organize a selection of approximately 75 articles chronologically ordered into a resource that belongs in the lounges and reading rooms of mathematics departments and mathematicians alike. The chronological ordering of articles may be displeasing to those who find the seemingly jumpy nature of the topics to be less than ideal. Personally, I found the presentation more like that of a Martin Gardner book which likewise typically consists of segments arising out of columns that may be seemingly unrelated to their immediate neighbours.

The authors (Guy, Gardner, Wagon, Dudley, Dunham, . . .) will be familiar to any reader of recreational mathematics or other material geared to undergraduate or senior secondary mathematical audiences. Anyone reading this review is bound to enjoy the offerings of this book. There really is no simple way to capture its scope and breadth. Readers familiar with *Math Horizons* will have a sense of what to expect; the rest of you may wish to avail of a genuine mathematical smorgasbord of ideas. The table of contents spans four pages. The opening article, entitled "John Horton Conway—Talking a Good Game" (reprinted from Spring 1994), and the closing article, "Knots to You" (reprinted from November 2003), surround an assortment of others, including: "Weird Dice", "The Instability of Democratic Decisions", "Was Gauss Smart?", "Egyptian Rope, Japanese Paper and High School Math", and "The World's First Mathematics Textbook".

Rarely have I seen such a wonderful assortment of mathematics displayed in such an accessible manner. This book would be a rich resource for mathematics clubs, budding secondary school mathematicians, or anyone further along in their mathematical journey. This may become a core reference in one of my future courses with prospective or practicing mathematics teachers, as it offers plenty of content along with insight into what doing mathematics is really about.