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SYNOPSIS

166 Butterfly Metamorphosis

by *Andy Liu*

The setting of the Butterfly Theorem involves three concurrent chords in a circle.

Butterfly Theorem. Let PQ, AB, and CD be three chords of a circle concurrent at a point M, with A and D on one side of PQ and B and C on the other side. If $PM = QM$, then $XM = YM$, where X and Y are the points of intersection of PQ with AC and BD, respectively.

The author's metamorphosis changes the setting to three concurrent cevians in a triangle. He then uses techniques developed in that context to give a simple proof of the Butterfly Theorem.

Enjoy!

169 Problems: 3226–3238

This month's "free sample" is:

3235. *Proposed by Geoffrey A. Kandall, Hamden, CT, USA.*

Let ABC be a triangle, and let A_1, B_1, C_1 be points on the sides BC, CA, AB, respectively, such that

$$\frac{BA_1}{A_1C} = \frac{CB_1}{B_1A} = \frac{AC_1}{C_1B} = k\,.$$

Let $\alpha = AA_1$, $\beta = BB_1$, $\gamma = CC_1$, and $\lambda = \dfrac{k^2+k+1}{(k+1)^2}$. Prove that

(a) $\alpha^2+\beta^2+\gamma^2 = \lambda(a^2+b^2+c^2)$;
(b) $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2 = \lambda^2\left(a^2b^2+b^2c^2+c^2a^2\right)$;
(c) $\alpha^4+\beta^4+\gamma^4 = \lambda^2\left(a^4+b^4+c^4\right)$.

. . . .

3235. *Proposé par Geoffrey A. Kandall, Hamden, CT, É-U.*

Soit respectivement A_1, B_1 et C_1 des points sur les côtés BC, CA et AB d'un triangle ABC, de sorte que

$$\frac{BA_1}{A_1C} = \frac{CB_1}{B_1A} = \frac{AC_1}{C_1B} = k\,.$$

Soit $\alpha = AA_1$, $\beta = BB_1$, $\gamma = CC_1$ et $\lambda = \dfrac{k^2+k+1}{(k+1)^2}$. Montrer que

(a) $\alpha^2+\beta^2+\gamma^2 = \lambda(a^2+b^2+c^2)$;
(b) $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2 = \lambda^2\left(a^2b^2+b^2c^2+c^2a^2\right)$;
(c) $\alpha^4+\beta^4+\gamma^4 = \lambda^2\left(a^4+b^4+c^4\right)$.

175 Solutions: 3125–3136