Mayhem Problems

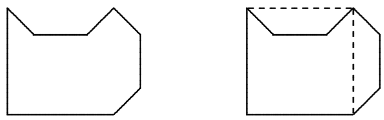
Please send your solutions to the problems in this edition by **1 August 2007**. *Solutions received after this date will only be considered if there is time before publication of the solutions.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

The editor thanks Jean-Marc Terrier and Martin Goldstein of the University of Montreal for translations of the problems.

M288. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

The following figure can be cut into two pieces and reassembled into a square, by simply cutting off the 'tab' and placing it in the cutaway at the top, as shown in the second image.



Determine a method to cut the given figure into three pieces which can be reassembled to form a square. (Find a method which is essentially different from cutting it into two pieces; for example, cutting the tab into two pieces would not be considered different from the two-piece dissection.)

M289. *Proposed by K.R.S. Sastry, Bangalore, India.*

Solve the following equation for real x:

$$\log\left(x+\sqrt{5x-\tfrac{13}{4}}\right) \;=\; -\log\left(x-\sqrt{5x-\tfrac{13}{4}}\right)\,.$$

M290. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

Give a purely geometric proof that $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$.

M291. *Proposed by Robert Bilinski, Collège Montmorency, Laval, QC.*

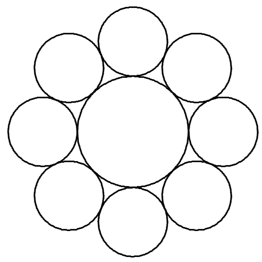
The right triangle having sides 3, $\sqrt{7}$, and 4, has the strange property that the two integer lengths sum to the value under the square root sign for the length of the third side.

1. Find another such triangle.
2. Prove that there are infinitely many such triangles, and show how to construct them.
3. Does the formula work only for integers?

M292. *Proposed by José Luis Díaz-Barrero, Universitat Politècnica de Catalunya, Barcelona, Spain.*

Let x be a positive number. Prove that $\sqrt{\frac{[x]}{x+\{x\}}}+\sqrt{\frac{\{x\}}{x+[x]}}>1$, where $[x]$ and $\{x\}$ represent the integer part and the fractional part of x, respectively.

M293. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*



Eight equal circles are mutually tangent in pairs and tangent externally to a unit circle. Determine the common radii of the eight smaller circles.

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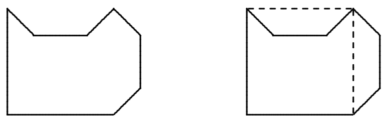
M288. *Proposé par Bruce Shawyer, Université Memorial de Terre-Neuve, St. John's, NL.*

La figure ci-dessous peut être coupée en deux morceaux qu'on peut réarranger pour former un carré, comme le montre le second dessin.



Trouver une méthode pour couper la figure donnée en trois morceaux pouvant former un carré par réarrangement. (Cette méthode doit être essentiellement différente de la première ; simplement couper en deux le morceau ajouté pour former le premier carré ne compte pas.)

M289. *Proposé par K.R.S. Sastry, Bangalore, Inde.*

Trouver les solutions réelles de l'équation :

$$\log\left(x+\sqrt{5x-\tfrac{13}{4}}\right) \;=\; -\log\left(x-\sqrt{5x-\tfrac{13}{4}}\right)\,.$$

M290. *Proposé par Bruce Shawyer, Université Memorial de Terre-Neuve, St. John's, NL.*

Trouver une démonstration purement géométrique de l'égalité $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$.

M291. *Proposé par Robert Bilinski, Collège Montmorency, Laval, QC.*

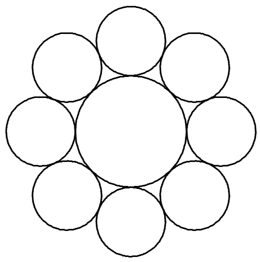
Le triangle rectangle de côtés 3, $\sqrt{7}$ et 4 possède la curieuse propriété qu'un de ses côtés est la racine carrée de la somme des côtés mesurés par des entiers.

1. Trouver un autre tel triangle.
2. Montrer qu'il existe une infinité de tels triangles et décrire leur construction.
3. La formule n'est-elle valable que pour des entiers ?

M292. *Proposé par José Luis Díaz-Barrero, Université Polytechnique de Catalogne, Barcelone, Espagne.*

Soit x un nombre positif. Montrer que $\sqrt{\dfrac{[x]}{x+\{x\}}}+\sqrt{\dfrac{\{x\}}{x+[x]}}>1$, où $[x]$ et $\{x\}$ désignent respectivement les parties entière et fractionnaire de x.

M293. *Proposé par Bruce Shawyer, Université Memorial de Terre-Neuve, St. John's, NL.*



On couronne le cercle unité avec huit petits cercles égaux tangents et tangents deux à deux. Trouver leur rayon commun.