

MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as a **Mathematical Journal for and by High School and University Students**. It continues, with the same emphasis, as an integral part of *Crux Mathematicorum with Mathematical Mayhem*.

The Mayhem Editor is Jeff Hooper (Acadia University). The Assistant Mayhem Editor is Ian VanderBurgh (University of Waterloo). The other staff members are John Grant McLoughlin (University of New Brunswick), Mark Bredin (St. John's-Ravenscourt School, Winnipeg), Monika Khbeis (Father Michael Goetz Secondary School, Mississauga), Eric Robert (Leo Hayes High School, Fredericton), Larry Rice (University of Waterloo), and Ron Lancaster (University of Toronto).

Mayhem Editorial

Jeff Hooper

Welcome to another year of Mathematical Mayhem! I must apologize for my greetings not appearing in the first issue of 2007. Although the first issue of *CRUX with MAYHEM* appears in February of each year, the editing of material for this issue actually occurs in November. With the changeover in duties and the usual rush of late fall, I simply missed doing this.

Before introducing myself as the new editor of Mathematical Mayhem, I wish to thank my predecessor, Shawn Godin. In the middle of last year, Shawn reluctantly decided to step down as Mayhem editor. Shawn is an Ottawa-area high school teacher and education consultant, who has managed to squeeze in all of his Mayhem duties on top of his already heavy schedule. He also recently decided to return to university to work toward his doctorate. With all of these commitments and a young family, Shawn felt that it was the proper time to step down.

It would be difficult to overstate Shawn's contribution to Mayhem. Over his years of involvement, he has kept Mayhem alive and true to its original purpose as a source of mathematical problems and problem-solving ideas suitable for high school students. We will continue down that road. We will miss you, Shawn, and we wish you well with your new challenges! You are welcome back anytime!

Now, a few words about your new Mayhem editor. I am an Associate Professor in the Department of Mathematics and Statistics at Acadia University in Nova Scotia. Before that, I spent several years at the Universities of Cambridge, Durham, and Waterloo. My main area of mathematical interest is number theory, but I also have an interest in mathematics education. This educational slant has led me down a number of other paths, including curriculum consulting for the Nova Scotia Department of Education and serving

as Nova Scotia's provincial coordinator for the Maritime Mathematics Competition written by regional high school students. Many problems from these competitions have appeared previously in the Skoliad. When not busy with all of these other things, I usually spend time with my children, or else play music (poorly).

Last fall the **CRUX with MAYHEM** Editorial Board began a discussion regarding the focus of *Pólya's Paragon*. When *Pólya's Paragon* was started a number of years ago by Paul Ottaway, there was a specific direction to the articles: they focused on a technique or idea that was important in problem-solving and then illustrated that technique on two or three problems. Since then, the content of the Paragon has varied quite a lot. We have decided to return to the original format for future Paragons. This means that we will not have a Paragon every month, but only when we find an article that fits the mold. However, we will continue to welcome articles. I wish to invite readers to submit to us short articles intended for a high school audience. If you feel they fit the mold for a Paragon article, be sure to point that out. We are also always in need of good Mayhem problems. If you can supply us with some, we would be most grateful.

In closing, let me thank all of you for the work you put into Mayhem, either directly or even by just reading and working the problems. I would be very happy to hear from you if you have comments or suggestions. I hope you enjoy the 2007 volume of Mayhem!

Mayhem Problems

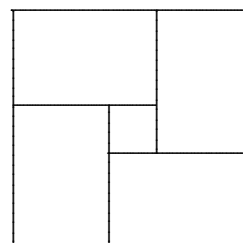
*Veillez nous transmettre vos solutions aux problèmes du présent numéro avant le **premier juillet 2007**. Les solutions reçues après cette date ne seront prises en compte que s'il nous reste du temps avant la publication des solutions.*

Chaque problème sera publié dans les deux langues officielles du Canada (anglais et français). Dans les numéros 1, 3, 5 et 7, l'anglais précédera le français, et dans les numéros 2, 4, 6 et 8, le français précédera l'anglais.

La rédaction souhaite remercier Jean-Marc Terrier et Martin Goldstein, de l'Université de Montréal, d'avoir traduit les problèmes.

M282. *Proposé par J. Walter Lynch, Athens, GA, USA.*

Quatre rectangles sont arrangés en un motif carré, de sorte qu'ils entourent un carré plus petit. Soit S l'aire du carré extérieur et Q celle du carré intérieur. Si $S/Q = 9 + 4\sqrt{5}$, déterminer le rapport des côtés des rectangles.



M283. *Proposé par Neven Jurič, Zagreb, Croatie.*

Trouver la relation entre x et y , si

$$x^2 + y \cos^2 \alpha = x \sin \alpha \cos \alpha \quad \text{et} \quad x \cos 2\alpha + y \sin 2\alpha = 0.$$

(On suppose que x et y sont tous deux non nuls.)

M284. *Proposé par Bruce Sawyer, Université Memorial de Terre-Neuve, St. John's, NL.*

Montrer que

$$\tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{1}{4} \right) + \tan^{-1} \left(\frac{1}{13} \right) = \frac{\pi}{4}.$$

M285. *Proposé par José Luis Díaz-Barrero, Université Polytechnique de Catalogne, Barcelone, Espagne.*

Soit a , b et c trois nombres strictement positifs tels que $a+b+c \geq 3abc$.
Montrer que $a^2 + b^2 + c^2 \geq 2abc$.

M286. *Proposé par K.R.S. Sastry, Bangalore, Inde.*

Si $xy + yz + zx = 1$, montrer que

$$(a) \quad \frac{x}{1+x^2} + \frac{y}{1+y^2} + \frac{z}{1+z^2} = \frac{2}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}};$$

$$(b) \quad \frac{x}{1+x^2} + \frac{y}{1+y^2} + \frac{z}{1+z^2} = \frac{2}{x+y+z-xyz}.$$

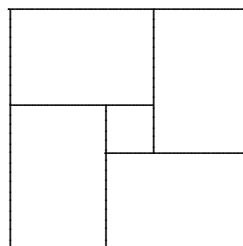
M287. *Proposé par Bruce Sawyer, Université Memorial de Terre-Neuve, St. John's, NL.*

Avec la règle et le compas, construire la moyenne harmonique de deux nombres réels donnés a et b .

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M282. *Proposed by J. Walter Lynch, Athens, GA, USA.*

Four rectangles are arranged in a square pattern so that they enclose a smaller square. Let S be the area of the outer square and Q the area of the inner square. If $S/Q = 9 + 4\sqrt{5}$, determine the ratio of the sides of the rectangles.



M283. *Proposed by Neven Jurič, Zagreb, Croatia.*

Determine the relationship between x and y if

$$x^2 + y \cos^2 \alpha = x \sin \alpha \cos \alpha \quad \text{and} \quad x \cos 2\alpha + y \sin 2\alpha = 0.$$

(Assume that both x and y are non-zero.)

M284. Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.

Prove that

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \frac{\pi}{4}.$$

M285. Proposed by José Luis Díaz-Barrero, Universitat Politècnica de Catalunya, Barcelona, Spain.

Let a , b , and c be strictly positive numbers such that $a + b + c \geq 3abc$. Prove that $a^2 + b^2 + c^2 \geq 2abc$.

M286. Proposed by K.R.S. Sastry, Bangalore, India.

If $xy + yz + zx = 1$, show that

$$(a) \frac{x}{1+x^2} + \frac{y}{1+y^2} + \frac{z}{1+z^2} = \frac{2}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}};$$

$$(b) \frac{x}{1+x^2} + \frac{y}{1+y^2} + \frac{z}{1+z^2} = \frac{2}{x+y+z-xyz}.$$

M287. Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.

Given two positive real numbers a and b , construct their harmonic mean with straightedge and compass.

Mayhem Solutions

M232. Proposé par Nicholas Buck, College of New Caledonia, Prince George, CB, et John Grant McLoughlin, Université du Nouveau-Brunswick, Fredericton, NB.

On peut recouvrir un échiquier standard de 8 lignes par 8 colonnes avec 32 dominos, chaque domino couvrant deux cases adjacentes. Supposons qu'on enlève au hasard deux cases. Si le nouvel échiquier obtenu ne peut plus être recouvert par 31 dominos, quelle est la probabilité pour que :

1. les deux cases enlevées soient dans la même ligne ?
2. les deux cases enlevées se touchent en un sommet (diagonalement, horizontalement ou verticalement) ?

Solution par Jean-David Houle, Cégep de Drummondville, Drummondville, QC.

Supposons, sans perte de généralité, que les cases de l'échiquier sont alternativement noires et blanches, comme celles d'un échiquier standard. On montre tout d'abord que le nouvel échiquier ne peut plus être recouvert par 31 dominos si et seulement si les deux cases retirées sont de la même couleur.

En effet, un domino recouvre deux cases adjacentes verticalement ou horizontalement, donc les deux cases sont toujours de couleurs différentes, soit une noire et une blanche. Si on recouvre l'échiquier par 31 dominos, il doit donc y avoir 31 cases blanches et 31 cases noires. Puisque l'échiquier original contient 32 cases de chaque couleur, les deux cases retirées doivent être de couleurs différentes. [*Réd* : De plus, si les deux cases retirées sont de couleurs différentes, elles sont aux coins opposés d'un rectangle de dimensions pair par impair et ce rectangle peut toujours être recouvert par des dominos. En séparant l'échiquier avec des tranches dans la direction de la dimension impaire du rectangle, on obtient des rectangles qui ont tous au moins une dimension paire et qui peuvent donc tous être recouverts avec des dominos.] Ceci indique que si on ne peut plus recouvrir l'échiquier par 31 dominos, les deux cases retirées sont de la même couleur.

1. Une ligne contient 4 cases de la même couleur, alors après avoir retiré la première case, il reste 3 cases de la même couleur sur la même ligne. Donc la probabilité est

$$\frac{\text{cas favorables}}{\text{cas possibles}} = \frac{3}{31} \approx 0,1935.$$

2. Deux cases qui se touchent en un sommet verticalement ou horizontalement sont nécessairement de couleurs différentes, donc les deux cases retirées se touchent diagonalement. On note que pour une couleur donnée, 18 cases ont 4 contacts diagonaux, 12 cases ont 2 contacts diagonaux et 2 cases ont un contact diagonal. Donc la probabilité est

$$\frac{\text{cas favorables}}{\text{cas possibles}} = \frac{18 \cdot 4 + 12 \cdot 2 + 2 \cdot 1}{32 \cdot 31} = \frac{98}{992} \approx 0,0988.$$

Autre solution soumise par Richard I. Hess, Rancho Palos Verdes, CA, É-U.

M233. *Proposed by Richard K. Guy, University of Calgary, Calgary, AB.*

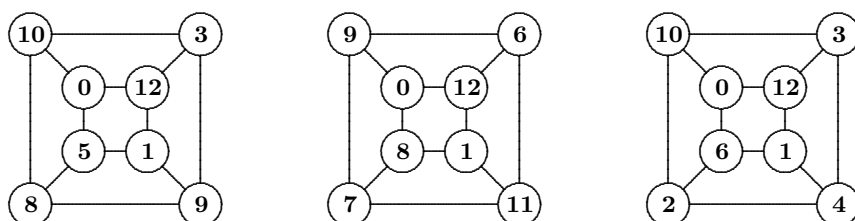
Can you place eight distinct integers selected from 0 to 12 at the vertices of a cube so that the twelve edges have the differences 1, 2, . . . , 12 between their end-points?

Either find a way to do this, or prove that it is impossible.

Solution by the proposer, modified by the editor.

Since 12 is one of the differences, the numbers 0 and 12 must be on two adjacent vertices of the cube. Since 11 is another difference, either a

vertex labelled 11 is adjacent to the vertex labelled 0 or a vertex labelled 1 is adjacent to the vertex labelled 12. However, these two possibilities are related by the mapping $f(n) = 12 - n$. Let us assume that a vertex labelled 1 is adjacent to the vertex labelled 12. Here are three such distinct cubes :



Also solved by Richard I. Hess, Rancho Palos Verdes, CA, USA. The proposer suspects that there may be as many as a dozen or more distinct solutions.

M234. *Proposé par K. R. S. Sastry, Bangalore, Inde.*

Soit J un nombre de deux chiffres sans communs diviseurs autres que 1. En permutant ces deux chiffres, on obtient un nombre I qui est $p\%$ plus grand que J . Trouver toutes les valeurs possibles de p , p étant un nombre naturel positif plus petit que 100.

Solution par Jean-David Houle, Cégep de Drummondville, Drummondville, QC, modifié par le rédacteur.

Soit $J = 10x + y$ où x et y sont des entiers entre 1 et 9, avec $(x, y) = 1$. Donc $I = 10y + x$. Aussi, $I = \left(1 + \frac{p}{100}\right) J$. En isolant p , on obtient :

$$p = 100 \cdot \frac{I - J}{J} = \frac{900(y - x)}{J}. \quad (1)$$

Puisque p est un nombre naturel, alors $y > x$ et $10x + y \mid 900(y - x)$.

Supposons que J et $y - x$ ont un diviseur commun plus grand que 1. Alors ils ont un diviseur principal commun q . Puisque $q \mid J = 10x + y$ et $q \mid y - x$, on a

$$q \mid (10x + y) - (y - x) = 11x.$$

Puisque $q < y - x \leq 9$, on obtient $q \mid x$. Donc $q \mid (y - x) + x = y$, qui est une contradiction, parce qu'ils n'ont aucun diviseur commun plus grand que 1. Ainsi, $(J, y - x) = 1$.

Donc $J \mid 900$. Les valeurs possibles pour J sont $\{12, 15, 18, 25, 45\}$. En remplaçant ces valeurs dans (1), on trouve $p \in \{75, 240, 350, 108, 20\}$. Puisque $p < 100$, on a $p = 20$ et $p = 75$.

En outre résolu par Michelle Ellenburg et Christopher Odom, étudiants, Angelo State University, San Angelo, TX, É-U; et Richard I. Hess, Rancho Palos Verdes, CA, É-U.

M235. *Proposé par Ron Lancaster, Université de Toronto, Toronto, ON.*

Résoudre l'équation

$$2^x + 2^{x+1} + \dots + 2^{x+2006} = 4^x + 4^{x+1} + \dots + 4^{x+2006}.$$

Solution par Jean-David Houle, Cégep de Drummondville, Drummondville, QC.

On simplifie l'équation :

$$\begin{aligned} 2^x(1 + 2^1 + \dots + 2^{2006}) &= 4^x(1 + 4^1 + \dots + 4^{2006}), \\ 1 + 2^1 + \dots + 2^{2006} &= 2^x(1 + 4^1 + \dots + 4^{2006}). \end{aligned}$$

On remplace les séries géométriques par leur sommes :

$$\begin{aligned} 1 + 2^1 + \dots + 2^{2006} &= \frac{1 - 2^{2007}}{1 - 2} = 2^{2007} - 1 \\ \text{et } 1 + 4^1 + \dots + 4^{2006} &= \frac{1 - 4^{2007}}{1 - 4} = \frac{4^{2007} - 1}{3}. \end{aligned}$$

Donc :

$$\begin{aligned} 1 + 2^1 + \dots + 2^{2006} &= 2^x(1 + 4^1 + \dots + 4^{2006}), \\ 2^{2007} - 1 &= 2^x \cdot \frac{4^{2007} - 1}{3}, \\ 2^x &= \frac{(3)(2^{2007} - 1)}{4^{2007} - 1} = \frac{(3)(2^{2007} - 1)}{(2^{2007} + 1)(2^{2007} - 1)} = \frac{3}{2^{2007} + 1}. \end{aligned}$$

On peut en approximer le résultat :

$$x = \log_2\left(\frac{3}{2^{2007} + 1}\right) \approx \log_2\left(\frac{3}{2^{2007}}\right) = \log_2 3 - 2007 \approx -2005,415.$$

En outre résolu par RICHARD I. HESS, Rancho Palos Verdes, CA, É-U; et JOSH TREJO et MANDY RODGERS, étudiants, Angelo State University, San Angelo, TX, É-U.

M236. *Proposed by Edward J. Barbeau, University of Toronto, Toronto, ON.*

A traveller to a strange island discovers that it is inhabited by knights who can only make true statements and knaves who can only make false statements. One day a traveller encountered three inhabitants, whom we will call *A*, *B*, and *C*, and asked, "How many knights are there among you three?"

A made an answer, which the traveller missed, but which was understood by the other two. When *B* was asked what *A* said, *B* responded, "A said that there is one knight among us."

"Don't believe *B*," exclaimed *C*, "he is lying."

What are *B* and *C*?

Solution by Mandy Rodgers and Josh Trejo, students, Angelo State University, San Angelo, TX, USA.

Since B and C made contradictory statements, they cannot both be knaves, or both knights. Thus, there are two cases to consider: B is a knight and C is a knave, or B is a knave and C is a knight.

Assume first that B is a knight and is telling the truth (and C is a knave). Then A really did say that there is one knight. Now, if A were a knight, his statement "There is one knight among us" would need to be true and would lead to a contradiction, since both A and B would be knights. If A were a knave, his statement "There is one knight among us" would need to be false, which would again lead to a contradiction, since B would be the one and only knight. Hence, B cannot be a knight.

Therefore, B is a knave and is telling a lie (and C is a knight), which answers the question asked. We should explore this possibility to see if it can actually occur. Assume that B is a knave and C is a knight. Then A did not say that there is one knight. Now, if A were a knight, he could have said that there were 2 knights, which would be consistent, since both A and C would be knights. If A were a knave, he could have said they were all knights or all knaves. Thus, although it is not possible to determine whether A is a knight or a knave, we do know that B is a knave and C is a knight.

Also solved by JEAN-DAVID HOULE, Cégep de Drummondville, Drummondville, QC; and MICHELLE ELLENBURG and CHRISTOPHER ODOM, students, Angelo State University, San Angelo, TX, USA.

M237. *Proposed by K.R.S. Sastry, Bangalore, India.*

Let ABC be an isosceles triangle with $AB = AC$, and let the lengths of the sides be integers with no common divisor other than 1. The incentre I divides the internal angle bisector AD such that $\frac{AI}{ID} = \frac{25}{24}$. Determine the radius of the incircle of $\triangle ABC$.

Solved by Richard I. Hess, Rancho Palos Verdes, CA, USA.

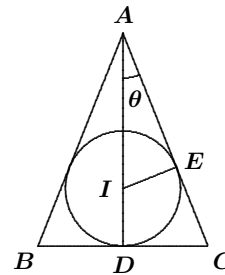
Let a and b be relatively prime integers such that $a = BC$ and $b = AC = AB$, and let $\theta = \angle DAC$. We know that $\sin \theta = \frac{1}{2}a/b$ and $\sin \theta = IE/AI$. Since $IE = ID$, we conclude that

$$\sin \theta = \frac{a}{2b} = \frac{24}{25}.$$

Hence, since a and b are relatively prime, we have $a = 48$ and $b = 25$. Then $EC = DC = \frac{1}{2}a = 24$; thus, $AE = AC - EC = 1$. Now,

$$\begin{aligned} r &= IE = AE \tan \theta = \tan \theta = \frac{\sin \theta}{\cos \theta} \\ &= \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} = \frac{24/25}{7/25} = \frac{24}{7}. \end{aligned}$$

There was one incorrect solution submitted.



Problem of the Month

Ian VanderBurgh

This month, we have a couple of problems demonstrating that knowing too much algebra may be dangerous!

Problem #1 (1974 Gauss Contest). A car is driven up a 1 km long hill at 30 km/h, and continues down the other side, which is also 1 km in length. The speed the car must be driven on the down slope, in km/h, in order to average 60 km/h for the whole trip is

- (A) 30 (B) 90 (C) 60 (D) 120 (E) none of these

It is tempting to answer 90 km/h, since the average of 30 and 90 is 60, but this somehow seems too easy. (To boot, you probably have that nagging voice in the back of your head reminding you of something your Grade 5 mathematics teacher told you about this sort of problem. . . .)

The most important thing to remember in solving this problem is that speed equals distance divided by time, which is the same as distance equals speed multiplied by time, or time equals distance divided by speed.

Solution #1: To drive up the 1 km hill at 30 km/h takes $\frac{1}{30}$ hour, or 2 minutes. To average 60 km/h over the whole 2-km trip, the total driving time must be $\frac{2}{60}$ hour, or 2 minutes. Then the downhill part of the trip must take $2 - 2 = 0$ minutes. Since this is not possible, the answer must be (E).

Now, that was a surprising answer!

If we know some algebra (and try to use it), the solution becomes a bit more complicated.

Solution #2: Suppose the car is driven down the hill at v km/h. To find the average speed, we find the total distance driven (2 km in this case) and divide by the total time. The time for the uphill portion is $\frac{1}{30}$ hour as in Solution #1 above. The time for the downhill portion is $1/v$ hour. Therefore, the total time is $\frac{1}{30} + \frac{1}{v}$ hour. Hence, the average speed, in km/h, is

$$\frac{2}{\frac{1}{30} + \frac{1}{v}} = 60,$$

which simplifies to

$$2 = 60 \left(\frac{1}{30} + \frac{1}{v} \right) = 2 + \frac{60}{v}.$$

Thus, $60/v = 0$, which is impossible.

Let's try a couple of variations of the problem. First, try the problem with 40 km/h instead of 30 km/h. Try it by both of the methods used above. Did you get 120 km/h? (Going uphill for 1 km at 40 km/h should take $\frac{1}{40}$ of an hour, which is $1\frac{1}{2}$ minutes, leaving half a minute out of the total of 2 minutes to drive the remaining 1 km downhill.)

What happens if we replace the 30 km/h with 20 km/h? If we proceed mathematically (without thinking), we find that the car must be driven down the hill at -60 km/h. One wonders what that really means! This shows that we always need to think about what we are doing.

Suppose the car is driven uphill at u km/h and then downhill at v km/h. What are the possible values of u that allow us to solve the problem (that is, to get a positive value for v)?

We model Solution #2 from above. The time to drive uphill is $1/u$ hours and to drive downhill is $1/v$ hours. Therefore, the total driving time is $(1/u) + (1/v)$ hours, and the average speed is

$$\frac{2}{\frac{1}{u} + \frac{1}{v}} = 60.$$

Now we solve for v in terms of u :

$$\begin{aligned} 2 &= 60 \left(\frac{1}{u} + \frac{1}{v} \right), \\ \frac{1}{30} - \frac{1}{u} &= \frac{1}{v}, \\ v &= \frac{1}{\frac{1}{30} - \frac{1}{u}} = \frac{30u}{u - 30}. \end{aligned}$$

We know that u is a positive real number. For v to be a positive real number, we need $u - 30 > 0$, or $u > 30$. Thus, an uphill speed of more than 30 km/h allows us to find a downhill speed that gives an average speed of 60 km/h.

For what *integer* values of u can we find a positive integer value of v that gives an average speed of 60 km/h? We know already that

$$v = \frac{30u}{u - 30} = \frac{30u - 900 + 900}{u - 30} = 30 + \frac{900}{u - 30}.$$

For v to be an integer, we need $\frac{900}{u - 30}$ to be an integer, and for this we need $u - 30$ to be a divisor of 900. You can list out the divisors of 900 and the corresponding values of u .

So, a problem that starts out being quite simple has lots of interesting ideas that can be gleaned from it. The most important thing to remember here is the very first simple solution. Thinking about this type of problem in a clever way will often get you the answer more easily than using a formal algebraic approach.

Let's try to apply this way of thinking to another problem.

Problem #2. Jeff is on a railway bridge joining A to B , and is $\frac{3}{8}$ of the way across from A . He hears a train approaching A ; it is travelling at 80 km/h. If he runs towards A , he will meet the train at A . If he runs towards B , the train will overtake him at B . How fast can he run?

Solution #1: Let's try this algebraically first. Suppose the bridge has a length R , the train is a distance d from point A , and Jeff's running speed is v . The amount of time it would take Jeff to run to A is $\frac{3}{8}R/v$ and to run to B is $\frac{5}{8}R/v$. (Each of these quantities is distance divided by speed.) The amount of time it would take the train to get to A is $d/80$ and to get to B is $(d + R)/80$. Since Jeff and the train would arrive at A or B at the same time,

$$\frac{\frac{3}{8}R}{v} = \frac{d}{80} \quad \text{and} \quad \frac{\frac{5}{8}R}{v} = \frac{d + R}{80}.$$

Now we have two equations with three unknowns and want to solve for v . Try fiddling with these before going on!

—Any luck? We could solve this by substituting one equation into the other, but here's a more clever way. By subtracting the first equation from the second one, we get

$$\frac{\frac{1}{4}R}{v} = \frac{R}{80},$$

which yields $v = 20$ km/h. Hence, Jeff runs at 20 km/h.

But perhaps we can find a better method. How about this?

Solution #2: In the amount of time that Jeff runs $\frac{3}{8}$ of the way across the bridge, the train gets to A . Suppose that Jeff runs $\frac{3}{8}$ of the way across the bridge towards B , not A . Once Jeff has run this distance, he is $\frac{3}{8} + \frac{3}{8} = \frac{3}{4}$ of the way from A to B , and the train is at A . But we also know that Jeff and the train get to B at the same time with Jeff running in this direction. Therefore, the train will travel the length of the entire bridge while Jeff runs the remaining $\frac{1}{4}$ of the length of the bridge. Thus, the train's speed is 4 times Jeff's speed; that is, Jeff's speed is $\frac{80}{4} = 20$ km/h.

Isn't that nice? To finish off, here is a problem of the same type for you to try:

A train passes completely through a tunnel in 10 minutes. A second train, twice as long, passes through the tunnel in 11 minutes. If both trains are travelling at the same speed, 72 km/h, determine the length of the tunnel and the lengths of the trains.