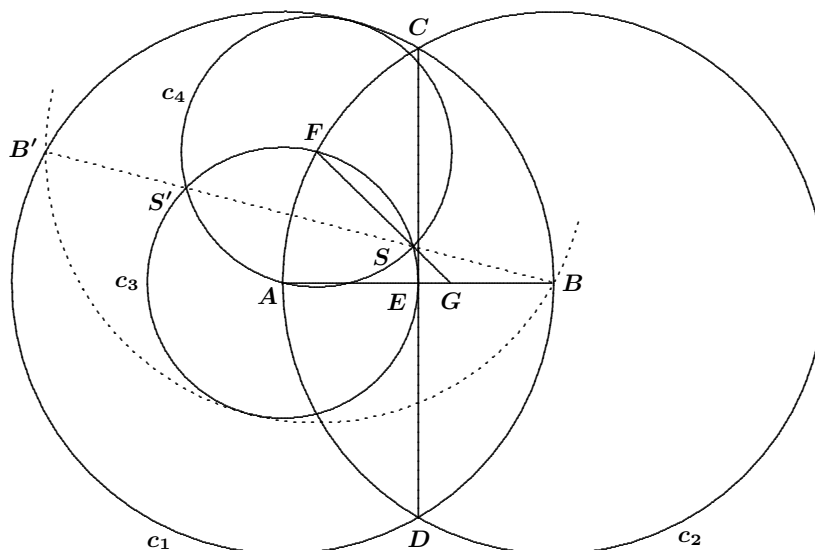


An Efficient Construction of the Golden Section

Kurt Hofstetter

We wish to divide efficiently, by ruler and compass, a given segment according to the golden section. We denote by $X(XY)$ the circle with X as centre and XY as radius. Let AB be the given segment.

1. Construct circle $c_1 = A(AB)$.
2. Construct circle $c_2 = B(AB)$. Let C and D be the points of intersection of c_1 and c_2 .
3. Draw line CD . Let CD intersect AB at E .
4. Construct circle $c_3 = A(AE)$. Let F be one of the points of intersection of c_3 and c_2 .
5. Construct circle $c_4 = F(AE)$. Let S and S' be the points of intersection of c_4 and c_3 such that FS extended will intersect the segment AB .
6. Extend FS to intersect AB at G .



Proposition. G divides AB according to the golden section.

Proof: Extend $S'B$ to intersect c_1 at B' . Because

$$AS = FS = AE = \frac{1}{2}AB = \frac{1}{2}FB,$$

we see that B and S are on the perpendicular bisector of FA , as are S' and B' . That is, S, S', B, B' lie on a line, and

$$AB = AB' = FB = FB' = 2AF.$$

Following the result in [1], we conclude that S divides $S'B$ according to the golden section. Finally, since $AS' = S'F = FS = SA$, we have $S'A$ parallel to $FS (= FG)$; hence, $S'S : SB = AG : GB$. Therefore, G divides AB according to the golden section.

Remarks.

1. This 6-step construction involves two appearances of the *vesica piscis*, which (according to the *Oxford English Dictionary*) is the name applied in the art world to the pointed oval region between the arcs of two equal circles, each through the centre of the other. Early artists employed the figure as an aureole enclosing the figure of Christ; it is also a common architectural feature. It seems somewhat ironic that this bit of Christian symbolism should be used to construct an object that was sacred to the Pythagoreans.

2. Reference [2] provides a 5-step division of a segment according to the golden section by ruler and rusty compass. The question remains, however, which construction is simpler?

References.

[1] K. Hofstetter, A Simple Construction of the Golden Section, *Forum Geom.* 2 (2002), 65–66.

<http://forumgeom.fau.edu/FG2002volume2/FG200208.pdf>

[2] K. Hofstetter, Division of a Segment in the Golden Section with Ruler and Rusty Compass, *Forum Geom.* 5 (2005), 135–136.

<http://forumgeom.fau.edu/FG2005volume5/FG200518.pdf>

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