BOOK REVIEW

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Index to Mathematical Problems 1975–1979
Edited by Stanley Rabinowitz and Mark Bowron, published by MathPro Press (Chelmsford, MA), 1999.
ISBN 0-9626401-2-3, hardcover, 8.5×11, x+518 pages, US$69.95.
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While there are standard ways to track down what is known in mathematical fields, it can be daunting to determine the history of a problem and its current status. Problems come in and out of fashion. They may be passed around by word of mouth, appear on contests in different places at different times, or be posed in any number of journals having problems sections. They may be picked up and abandoned, and their solutions (if any) communicated in various ways. Accordingly, it is difficult to discover their provenance, study their development, and assess their novelty. Many collections of problems have appeared in recent decades, particularly in Eastern Europe, but often these are collections from particular contests or reflect the idiosyncracies of the compilers of the collection. A reliable guide to more recent and notorious problems is Martin Gardner, whose *Scientific American* articles and many books cover a lot of territory. However, few of these provide a thematic index to problems and none are comprehensive enough to serve as a definitive guide to the literature. We have had to rely on a few individuals with a lot of experience and prodigious memories to impose any kind of order on the corpus of problems.

One such person is Stanley Rabinowitz, who received his doctorate in convexity, combinatorics, and number theory from the Polytechnic University of New York and is a software engineer and computer consultant. In 1989, he founded *MathPro Press* to produce problem indexes as well as collections of problems, compendia of mathematical results, and books on the use of computers in the solution of mathematical problems. Rabinowitz became convinced of the necessity of a source work to which problemists could turn for material and information on the origin and status of problems they come across. But how was he to deal with the chaotic wealth of material? His solution was to list all those problems that appeared in a selection of popular journals and contests and provide tools for searchers to find their way around. In 1992, he broke new ground with his publication of the *Index to Mathematical Problems 1980–1984*, which listed and classified all the problems from standard sources published in the English language during those years. This achievement was well received by the community, which eagerly awaited its successors. Finally, in 1999, the present volume was published with the collaboration of Mark Bowron, whose day job is driving an 18–wheeler transport truck.

The core of the book is the *Subject Index* (SI), 256 pages with the texts of about 4000 problems, listed by subject according to 17 classifications with many subheadings. Each problem appears with its author and source identification. The SI is an essentially complete collection of problems from 23 journals, mostly North American, and 6 competitions (Olympiads from Canada, USA, Australia, along with the International Mathematical Olympiads, and the Putnam and Kürchák contests). Canada is represented by the *Canadian Mathematical Bulletin* (which used to have a problem section), ***Crux Mathematicorum***, the *Ontario Mathematics Gazette*, and the *Ontario Secondary School Mathematics Bulletin*.

This is supplemented by other indexes:

- *Subject classification scheme*: list of titles in the SI with page references to the SI;
- *Problem locator*: list of journals and problem numbers with page references to the SI;
- *Problem chronology*: list of journals and problems, along with references to external sources with solutions and comments;
- *Author index*, with a key to pseudonyms;
- *Title index*, based on titles or topics assigned by journal editors;
- *Journal issue checklist*: details about publishers, problems editors, along with lists of problems according to issue;
- *Unsolved problems*: eight pages with complete statements;
- Citation index: references made in the period 1975–1979 to current and previously published problems in the journals;
- *Bibliography*;
- *Keyword index*.

Many readers will be drawn to the unsolved problems. Some of these are well known, such as the Collatz or $3x + 1$ problem (CRUX problem #133 [1976 : 67]). Some look tedious, technical or deep; others are more enticing. For example:

- Suppose that each square of an $n \times n$ chessboard is coloured either black or white. A square, formed by horizontal and vertical lines of the board, will be called chromatic if its four distinct corner squares are all of the same colour. Find the smallest n such that, with any such colouring, every $n \times n$ board must contain a chromatic square. (*American Mathematical Monthly* #6211)

- Let A and B be the unique non-decreasing sequences of odd integers and even integers, respectively, such that, for all $n \geq 1$, the number of integers i satisfying $A_i = 2n - 1$ is A_n and the number of integers i satisfying $B_i = 2n$ is B_n. That is,

$$A = \{1, 3, 3, 3, 5, 5, 5, 7, 7, 7, 9, 9, 9, 9, 9, \ldots\}$$

and

$$B = \{2, 2, 4, 4, 6, 6, 6, 6, 8, 8, 8, 8, \ldots\}.$$

Is the difference $|A_n - B_n|$ bounded? (*Mathematics Magazine* #1073)

Then there is this one from Paul Erdős:

- Find a sequence of positive integers $1 \leq a_1 < a_2 < a_3 < \cdots$ that omits infinitely many integers from every arithmetic progression (in fact, it has density 0), but which contains all but a finite number of terms of every geometric progression. Prove also that there is a set S of real numbers which omits infinitely many terms of any arithmetic progression, but contains every geometric progression (disregarding a finite number of terms). (*Pi Mu Epsilon Journal* #389)

The scholarship underpinning this volume is impressive. The editors have performed the difficult task of classifying the problems, matched up various versions of names and pseudonyms, tracked down if and where solutions may be found, and checked the accuracy of the statement of problems and the information provided about them. But the collection scoops up indiscriminately from its sources, so there is quite a mixed bag of problems. They may be fascinating and ingenious or banal, very difficult or trivially easy, significant challenges or dull exercises, novel or trite, narrowly specialized or broadly appealing, memorable or ephemeral, singular or type-cast. They can be solved by systematic techniques or through the right perspective and strokes of inspiration. Of course, the book does not offer guidance and the reader just has to dive in and try the problems. However, a teacher on the lookout for material suitable for various situations will find her efforts rewarded by problems of the right level of difficulty.

The long term viability of Rabinowitz's project is questionable. Problems are pouring into the literature regularly, many of which are neither novel nor particularly inventive. It is clear that the influx of new material is bound to outrun the capacity of the most energetic publisher to collect and classify it. Only the Internet can begin to keep on top of this task, and it may be that future books will have to focus on following the evolution of the most popular, interesting or elegant problems. However, journals with problem sections can help out with regular indexes of their own collections.

Rabinowitz has provided many resources on the Internet. A visit to `http://www.problemcorner.org` will provide you with 20000 problems that can be searched by keyword or author, and for which references to solutions in the literature are given. His main website (`http://www.mathpropress.com`) contains lists of problem books published during the last two centuries, all problem books published recently, web sites with problems from local, regional, and international competitions, 32 books with solutions to IMO problems, journals with problems sections, and mathematical dictionaries.