Pólya's Paragon

Playing Games with Mathematics (Part I)

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Recreational mathematics provides a wonderful vehicle for developing mathematical thinking. Game-playing arouses curiosity about the principles underlying the structures and strategies associated with the games. This is particularly evident when the players are students at a mathematical event such as the University of New Brunswick Math Camp. This column and its successor are based on a recent contribution to that event, in May 2006. Four of the games and challenges that were posed at the camp are shared here, followed by an additional problem involving polynomials.

The purpose of the challenges is threefold: to broaden one's knowledge of recreational mathematics; to engage in playful mathematical activity; and to develop mathematical arguments and proofs in response to the challenges. This latter point will be the focus of Part II, in which these challenges are to be discussed. It should be noted that this two-part model was used at the camp also. The challenges were shared one afternoon in an introductory session, and the discussion of the mathematical principles, including insights gained from participating students, took place the next evening.

1. Sim

Six dots are drawn on a piece of paper to form the vertices of a hexagon. Two players are each assigned a colour. The players take turns joining any two of the dots with a line segment, using their assigned colours. The loser is the player who completes a triangle with three of the original six dots as its vertices and with all three edges the same colour.

Challenge: Prove that there must always be a loser (and a winner).

2. 31

This mental math game involves a running total which starts at zero. Each player has the choice to add 1, 2, 3, 4, 5, or 6 to the total. Players alternate turns. The winner is the player who is able to bring the total to 31.

For example, player B is the winner in the following game:

Player	A	B	A	B	A	B	A	B
Chosen number	6	4	2	4	4	1	5	5
Total	6	10	12	16	20	21	26	31

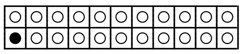
Challenge: Determine a winning strategy. You may choose to play first or second.

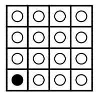
3. Chomp

Counters are placed in a rectangular grid such that one counter appears in each small rectangle. The counter in the bottom left-hand corner is a different colour than the others. Players take turns selecting one counter. If the counter selected occupies the bottom left-hand corner of a rectangle on the grid, all the counters in that rectangle are removed. The object is to force your opponent to select the differently coloured counter (the one in the bottom left-hand corner).

Try playing this game with rectangular boards of different sizes.

Challenge: Suppose that you play two games of Chomp in which the boards are $2 \times n$ and $k \times k$, examples of which are shown. Determine a winning strategy in each case. You may choose to play first or second.





4. Fifteen Finesse

The numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 are available for use in this game. Each number can be used only once. Two players alternate turns selecting one of the available numbers. To win the game, a player must obtain exactly three numbers that sum to 15. (Neither a pair of numbers, such as 7 and 8, nor a set of four numbers, such as 1, 3, 5, and 6, constitutes a winning combination.) The game ends in a draw if no player is able to acquire three numbers that sum to 15.

Challenge: Explain the underlying structure of the game, and suggest strategies that may help to win a game.

5. A Polynomial in Transition

Consider the polynomial $x^2 + 10x + 20$. Under the conditions below, is it possible to convert this polynomial to $x^2 + 20x + 10$? Justify your answer.

Conditions:

(i) On each step you may only change the constant term or the coefficient of x (but not both).

(ii) The change must be an increase of 1 or a decrease of 1.

(iii) The change must NOT produce a polynomial that can be factored into the form $(x+m)(x+n)$ where m and n are integers. For example, you could not begin by reducing 10 to 9, since $x^2+9x+20 = (x+5)(x+4)$.

Various sources have contributed ideas/games for this presentation, including books by Brian Bolt, Ian Stewart, Martin Gardner, and others. The polynomial problem is due to Ed Barbeau. A bibliography will be included in Part II. As usual, comments on any aspect of Pólya's Paragon are welcomed.