BOOK REVIEWS

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Mathematical Miniatures
By Svetoslav Savchev and Titu Andreescu, published by the Mathematical Association of America, 2003
ISBN 0-88385-645-X (paper), 230 pages, US$31.50.
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This book is a collection of problems from a variety of sources, such as national Olympiads and the Russian journal *Kvant*. The overall level is that of Olympiad problems, but many easier problems are included. Most importantly, the problems are chosen for their broad appeal and are grouped together according to technique or theme-and-variations. The style is nicely educational, and I found the book a pleasure to read.

One of my favorites in the book is Problem 23: Arrange the integers from 1 to n in a row so that the average of any two does not lie between them. The authors call this "an old and popular problem", but I had not heard of it before. It is challenging but not too difficult. What is noteworthy is that a Bulgarian high-school teacher came up with a two-dimensional variation. It is not obvious how to phrase such an extension. The result that was finally proved is that there are infinitely many integers n such that the integers from 1 to n^2 can be arranged in a square array so that the average of any two is not contained in the rectangle spanned by the two. The book is full of such extensions, which makes for interesting reading.

We all know that problem posers often have only one solution to a problem, but then learn of a much simpler solution from a student or journal contributor. A nice example occurs in Chapter 42, where it is related that Sergei Konyagin found a remarkably simple construction of an infinite set of positive integers such that the sum of any finite subset is not a perfect power. The problem poser had a quite complicated solution.

There is one problem in the book that I would consider... well... problematic. Problem 5 asks for a certain real solution to the equation $6x + 8\sqrt{1-x^2} = 5\sqrt{1+x} + \sqrt{1-x}$. But algorithms for doing this sort of thing have been known for centuries, and now such problems can be done in an eyeblink by a computer algebra system. Some of these algorithms are very sophisticated (for example, using cylindric algebras to verify inequalities). Knowledge of them (and their limitations) is critical to appreciating what is possible in modern mathematics. Thus, it seems sad that the problem community ignores their existence.

The book includes nine sets of three "Coffee Break Problems". These are not as hard as the main problems, and all have quite short solutions. But there are some gems in this group as well.

In all, this is an excellent collection, and any problemist will enjoy studying the contents and sharing them with his or her students.