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SYNOPSIS

193 Skoliad: No. 77 *Shawn Godin*

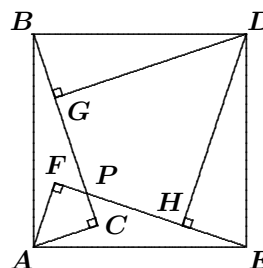
2003 Concours Hypatie (11^e année) / Hypatia Contest (Grade 11)

197 Mathematical Mayhem

197 Mayhem Problems: M144–M150

M144. *Proposé par Bruce Sawyer, Université Memorial de Terre-Neuve, St. John's, NL.*

Sur l'hypoténuse AB d'un triangle rectangle ABC on dessine un carré $ABDE$ de manière que C en soit un point intérieur. On dessine ensuite un triangle rectangle directement semblable BDG de sorte que G soit aussi un point intérieur du carré. On dessine finalement deux triangles rectangles indirectement semblables EDH et AEF , tels que H et F soient des points intérieurs du carré. Soit P le point d'intersection de BC et EF . Déterminer l'aire du quadrilatère $DGPH$ en fonction des côtés CA et CB du triangle rectangle original.



M145. *Proposé par Ovidiu-Gabriel Dinu, Balcesti-Valcea, Romania.*

Trouver tous les nombres naturels n pour lesquels n , $n + 2$, $n + 6$, $n + 8$ et $n + 14$ sont premiers.

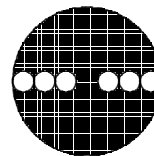
M146. *Proposé par Mohammed Aassila, Strasbourg, France.*

Soit a , b et c trois nombres positifs satisfaisant $a + b + c = 1$. Montrer que

$$(ab)^{5/4} + (bc)^{5/4} + (ca)^{5/4} < \frac{1}{4}.$$

M147. *Proposé par l'Équipe de Mayhem.*

Le diamètre d'un grand cercle est divisé en n parties égales pour construire n cercles plus petits, comme dans la figure. Trouver n , sachant que le rapport entre l'aire achurée et l'aire non achurée du grand cercle est $3 : 1$.



M148. *Proposé par Vedula N. Murty, Dover, PA, USA.*

Soit $x > 1$, $y > 1$, $z > 1$ et $x^2 = yz$. Trouver la valeur de

$$(\log_{zx} xy^4z) (\log_{xy} xyz^4) .$$

M149. *Proposé par Bruce Sawyer, Université Memorial de Terre-Neuve, St. John's, NL.*

Soit ABC un triangle rectangle de Héron possédant la propriété suivante : son aire est λ fois son périmètre, avec λ un entier positif. Trouver toutes les solutions (a, b, λ) . (Un triangle de Héron est un triangle dont la longueur des côtés et l'aire sont des nombres entiers.)

M150. *Proposé par Arkady Alt, San Jose, CA, USA.*

Soit deux nombres complexes z_1 et z_2 satisfaisant les conditions

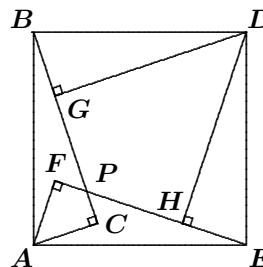
$$\begin{aligned} z_1 + z_2 &= -(i + 1) \\ z_1 \cdot z_2 &= -i \end{aligned}$$

Sans calculer z_1 et z_2 , trouver $z_1 \cdot \overline{z_2}$.

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M144. *Proposed by Bruce Sawyer, Memorial University of Newfoundland, St. John's, NL.*

A square $ABDE$ is drawn on the hypotenuse AB of right triangle ABC so that C lies in the interior of the square. A directly similar right triangle BDG is drawn so that G lies in the interior of the square. Indirectly similar right triangles EDH and AEF are drawn so that H and F lie in the interior of the square. Let BC and EF intersect at P . Determine the area of quadrilateral $DGPH$ in terms of the legs CA and CB of the original right triangle.



M145. Proposed by Ovidiu-Gabriel Dinu, Balcesti-Valcea, Romania.

Find all natural numbers n for which n , $n + 2$, $n + 6$, $n + 8$, and $n + 14$ are prime numbers.

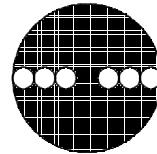
M146. Proposed by Mohammed Aassila, Strasbourg, France.

Let a , b , c be three positive numbers satisfying $a + b + c = 1$. Prove that

$$(ab)^{5/4} + (bc)^{5/4} + (ca)^{5/4} < \frac{1}{4}.$$

M147. Proposed by the Mayhem staff.

The diameter of a large circle is broken into n equal parts to construct n smaller circles, as shown. Determine n so that the ratio of the shaded area to the unshaded area in the large circle is $3 : 1$.



M148. Proposed by Vedula N. Murty, Dover, PA, USA.

Let $x > 1$, $y > 1$, $z > 1$ and $x^2 = yz$. Determine the value of

$$(\log_{zx} xy^4z) (\log_{xy} xyz^4) .$$

M149. Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.

A right-angled Heron triangle ABC has the following property: the area is λ times the perimeter, where λ is a positive integer. Determine all solutions (a, b, λ) . (A Heron triangle is a triangle with integer sides and integer area.)

M150. Proposed by Arkady Alt, San Jose, CA, USA.

Let two complex numbers z_1 and z_2 satisfy the conditions

$$\begin{aligned} z_1 + z_2 &= -(i + 1) \\ z_1 \cdot z_2 &= -i \end{aligned}$$

Without calculating z_1 and z_2 , find $z_1 \cdot \overline{z_2}$.

202 The Olympiad Corner: No. 238 *R.E. Woodrow*

Featuring the XXXVI Spanish Mathematical Olympiad National Round; Taiwan (ROC) Mathematical Olympiad 2000; 2000 Hungarian National Olympiad; 2000 Křschák; correction to Problem #6 of the 2001 Hungary-Israel Binational Mathematics Competition; readers' solutions to some of the problems of

- the Vietnamese Mathematical Olympiad 1999, Category A and B;
- the 16th Balkan Mathematical Olympiad.

221 Book Review *John Grant McLoughlin*

221 *A Friendly Mathematics Competiton: 35 Years of Teamwork in Indiana*

edited by Rick Gillman

Reviewed by John Grant McLoughlin

223 A Maximum Vertical Angle I *G.D. Chakerian and M.S. Klamkin*

Problem 414 proposed by C.N. Schmall [Amer. Math. Monthly (1917), 185–186] was to show that among all spherical triangles (convex) having the same base and equal altitudes, the isosceles triangle has the greatest vertical angle, and also to show that this was true for planar triangles.

The authors present a solution for the planar case, and then use that as a model for the proof in the spherical case.

227 A Maximum Vertical Angle II *D. Ruoff and J.C. Fisher*

The authors prove the same result as in I above, but their proof is done in absolute geometry

Enjoy comapring the two approaches!

229 Problems: 2939—2950

This month's "free sample" is:

2942. [2002–135] *Proposé par Toshio Seimiya, Kawasaki, Japon.*

Soit ABC un triangle tel que l'angle ABC soit le double de l'angle ACB . De plus, soit D un point sur le rayon CB tel que l'angle ADC soit la moitié de l'angle BAC . Montrer que

$$\frac{1}{CD} = \frac{1}{AB} - \frac{1}{AC}.$$

.....

Given $\triangle ABC$ with $\angle ABC = 2\angle ACB$, suppose that D is a point on the ray CB such that $\angle ADC = \frac{1}{2}\angle BAC$. Prove that

$$\frac{1}{CD} = \frac{1}{AB} - \frac{1}{AC}.$$

235 Solutions: 2548, 2799, 2839–2848