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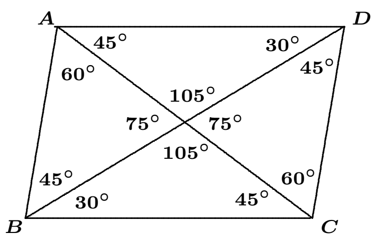
SYNOPSIS

M132. *Proposé par Peter Y. Woo, Biola University, La Mirada, CA, USA.*

(a) Qui a-t-il de faux dans le dessin du parallélogramme ci-contre? Les angles sont mesurés en degrés.

(b) Proposez une modification qui rendrait le dessin plausible. On ne doit pas modifier les segments issus de B.



M133. *Proposé par K.R.S. Sastry, Bangalore, Inde.*

Dans un pentagone $ABCDE$, chaque côté est parallèle à une diagonale. Montrer que le rapport d'une diagonale au côté parallèle correspondant est constant. En fait, cette constante est le nombre d'or. (Un tel pentagone est appelé *un pentagone d'or*.)

M134. *Proposé par K.R.S. Sastry, Bangalore, Inde.*

Dans un pentagone d'or $ABCDE$ (voir le problème précédent pour la définition), l'angle EAB est égal à l'angle BCD. Montrer que l'angle CDE est égal à l'angle DEA.

M135. *Proposé par l'Équipe de Mayhem.*

Trouver tous les nombres de deux chiffres avec exactement 8 diviseurs positifs.

M136. *Proposé par l'Équipe de Mayhem.*

Pour construire un nombre de cinq chiffres, on utilise chacun des chiffres 1, 4, 5, 7, et 8 une seule fois. Déterminer la somme de tous les différents nombres de cinq chiffres ainsi construits.

M137. *Proposé par Babis Stergiou, Lycio Psachnon Evias, Grèce.*

Soit a, b, $c > 0$, $a + b + c = 3$, et $abc = 1$.

(a) Montrer que $(a^2 + b)(a + b^2) \geq (a + a^2)(b + b^2)$;

(b) Déduire de (a) que (ou sinon montrer que)

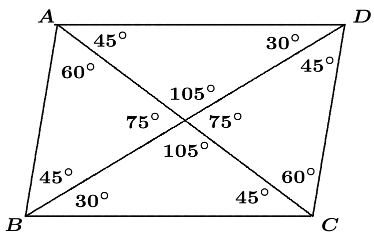
$$\frac{ab}{(a^2+b)(a+b^2)} + \frac{bc}{(b^2+c)(b+c^2)} + \frac{ca}{(c^2+a)(c+a^2)} \leq \frac{3}{4}.$$

. . . .

M132. *Proposed by Peter Y. Woo, Biola University, La Mirada, CA, USA.*

(a) What is wrong with the diagram of the parallelogram? The angle measures are in degrees.

(b) Suggest a modification that would make the diagram plausible. You are not allowed to modify the three lines through B.



M133. *Proposed by K.R.S. Sastry, Bangalore, India.*

In pentagon $ABCDE$, each side is parallel to a diagonal. Show that the ratio of a diagonal to the corresponding parallel side is constant. In fact, this constant is the golden ratio. (Such a pentagon is called a *golden pentagon*.)

M134. *Proposed by K.R.S. Sastry, Bangalore, India.*

In golden pentagon $ABCDE$ (see the preceding problem for the definition), we have $\angle EAB = \angle BCD$. Show that $\angle CDE = \angle DEA$.

M135. *Proposed by the Mayhem staff.*

Find all two-digit numbers with exactly 8 positive divisors.

M136. *Proposed by the Mayhem staff.*

The digits 1, 4, 5, 7, and 8 are each used once to form a five-digit number. Determine the sum of all such distinct five-digit numbers.

M137. *Proposed by Babis Stergiou, Lycio Psachnon Evias, Greece.*

Suppose a, b, $c > 0$, $a + b + c = 3$, and $abc = 1$.

(a) Prove that $(a^2 + b)(a + b^2) \geq (a + a^2)(b + b^2)$.

(b) Hence, or otherwise, prove that

$$\frac{ab}{(a^2+b)(a+b^2)} + \frac{bc}{(b^2+c)(b+c^2)} + \frac{ca}{(c^2+a)(c+a^2)} \leq \frac{3}{4}\,.$$

Problems about birthday probabilities are among the most interesting problems in probability due to their surprising answers, and they are more germane to the beginning probability student than many of the other problems in elementary probability texts. We will discuss three of the many birthday problems in existence.

Enjoy!

Featuring the Hungary-Israel Binational Mathematical Competition 2001 (Individual and Team Competition); Second Honh Kong (China) Mathematical Olympiad 1999; 17th Balkan Mathematical Olympiad 2000; readers' solutions to some of the problems of

- the Hungary-Israel Mathematical Competition 1999;
- the 12th Korean Mathematical Olympiad, Final Round, 1999;
- the Grossman Memorial Mathematical Olympiad 1999.

102 Some Necessary Conditions for a Real Polynomial to have only Real Roots *C.H. Harris Kwong and Amitabha Tripathi*

The purpose of this note is to derive some necessary conditions for a real polynomial of degree greater than one to have all its roots real. While there are several articles that deal with related problems, the results presented here are simple and elementary.

For example, the first result established is the following:

Theorem. Let $n \geq 2$, and let a_0, a_1, $\ldots$, a_n be real numbers with $a_n \neq 0$. If all roots of

$$p(x) \;=\; a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

are real, then $(n-1)a_1^2 \geq 2na_0a_2$.

Read on!

106 Problems: 2914—2925

This month's "free sample" is:

2914. *Proposé par Toshio Seimiya, Kawasaki, Japon.*

Sur les côtés d'un triangle acutangle ABC, on construit extérieurement des triangles isocèles de même type, DBC, ECA et FAB, de sorte que

$$\begin{aligned} \angle DBC \;&=\; \angle DCB \;=\; \angle EAC \;=\; \angle ECA \\ &=\; \angle FAB \;=\; \angle FBA \;=\; \angle BAC\,. \end{aligned}$$

Soit M le point milieu de BC, P et Q les intersections respectives de DE avec AC et de DF avec AB.

Montrer que $MP : MQ \;=\; AB : AC$.

. . . .

On the sides of an acute-angled triangle ABC, similar isosceles triangles DBC, ECA, FAB are constructed externally, such that

$$\begin{aligned} \angle DBC \;&=\; \angle DCB \;=\; \angle EAC \;=\; \angle ECA \\ &=\; \angle FAB \;=\; \angle FBA \;=\; \angle BAC\,. \end{aligned}$$

Let M be the mid-point of BC, and let P and Q be the intersections of DE with AC and of DF with AB, respectively.

Prove that $MP : MQ \;=\; AB : AC$.

112 Solutions: 2814–2825