THE SKOLIAD CORNER

No. 55

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Welcome to the Skoliad Corner. I would like to thank Robert Woodrow for his work over the years since he started Skoliad. I hope that I can do justice to his creation.

The format of Skoliad will remain very much the same; the only difference will be the time lag between the contests and solutions. We would like to print solutions from readers, in particular, students in high school or elementary school. Please send any solutions to

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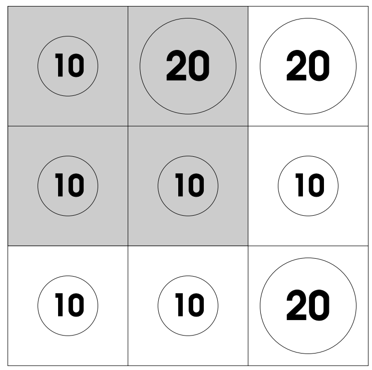
Please send your solutions to the problems in this issue by *1 January 2002*. Look for prizes for solutions in the new year.

The first entry to the corner is the 2000 National Bank Junior Mathematics Competition. The contest is written by students in years 9 to 11 and is organized by the University of Otago in Dunedin, New Zealand. My thanks go out to Warren Palmer and Derek Holton for forwarding the contest material to me. If you would like information on the contest feel free to contact them at

`nbjmc@maths.otago.ac.nz`

2000 National Bank Junior Mathematics Competition

1. In this problem we will be placing various arrangements of $10¢$ and $20¢$ coins on the nine squares of a 3×3 grid. Exactly one coin will be placed in each of the nine squares. The grid has four 2×2 subsquares each containing a corner, the centre, and the two squares adjacent to these. One example is shown in the diagram.



A 3×3 grid with the top left subsquare shaded in. This subsquare contains a total of $50¢$, while the others contain $60¢$, $40¢$, and $50¢$ respectively.

(a) Find an arrangement where the totals of the four 2×2 subsquares are $40¢$, $60¢$, $60¢$ and $70¢$ in any order.

(b) Find an arrangement where the totals of the four 2×2 subsquares are $50¢$, $60¢$, $70¢$ and $80¢$ in any order.

For each part of the problem below illustrate your answer with a suitable arrangement and an explanation of why no other suitable arrangement contains a larger (part(c)) or a smaller (part(d)) amount of money.

(c) What is the maximum amount of money which can be placed on the grid so that each of the 2×2 subsquares contains exactly $50¢$?

(d) What is the minimum amount of money which can be placed on the grid so that the average amount of money in each of the 2×2 subsquares is exactly $60¢$?

2. *Note: In this question an "equal division" is one where the total* **weight** *of the two parts is the same.*

(a) Belinda and Charles are burglars. Among the loot from their latest caper is a set of 12 gold weights of 1g, 2g, 3g, and so on, through to 12g. Can they divide the weights equally between them? If so, explain how they can do it; if not, why not?

(b) When Belinda and Charles take the remainder of the loot to Freddy the fence, he demands the 12g weight as his payment. Can Belinda and Charles divide the remaining 11 weights equally between them? If so, explain how they can do it; if not, why not?

(c) Belinda and Charles also have a set of 150 silver weights of 1g, 2g, 3g, and so on, through to 150g. Can they divide these weights equally between them? If so, explain how they can do it; if not, why not?

3. Humankind was recently contacted by three alien races: the Kweens, the Ozdaks, and the Merkuns. Little is known about these races except:

- Kweens always speak the truth.
- Ozdaks always lie.
- In any group of aliens a Merkun will never speak first. When it does speak, it tells the truth if the previous statement was a lie, and lies if the previous statement was truthful.

Although the aliens can readily tell one another apart, of course to humans all aliens look the same.

A high-level delegation of three aliens has been sent to Earth to negotiate our fate. Among them is at least one Kween. On arrival they make the following statements (in order):

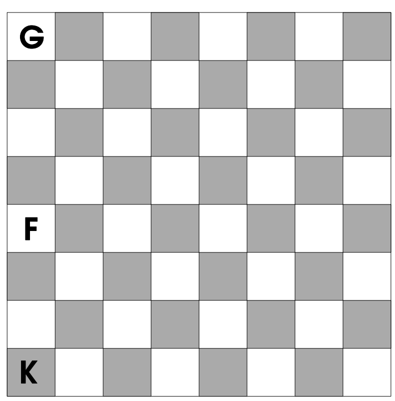
Statement A (First Alien): The second alien is a Merkun.

Statement B (Second Alien): The third alien is not a Merkun.

Statement C (Third Alien): The first alien is a Merkun.

Which alien or aliens can you be certain are Kween?

4. A chessboard is an 8×8 grid of squares. One of the chess pieces, the king, moves one square at a time in any direction, including diagonally.

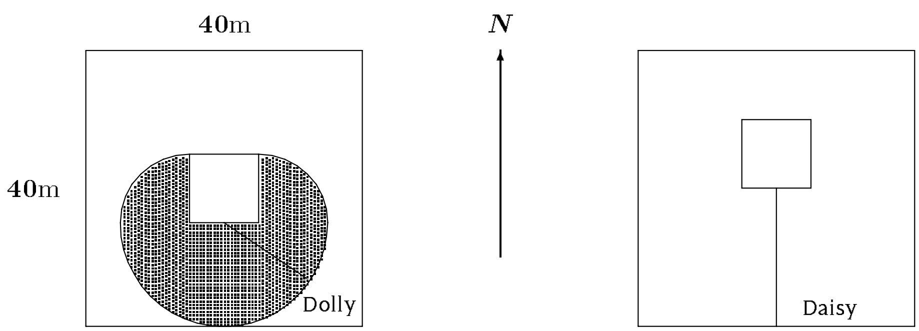


(a) A king stands on the lower left corner of a chessboard (marked **K**). It has to reach the square marked **F** in exactly 3 moves. Show that the king can do this in exactly **four** different ways.

(b) Assume that the king is placed back on the bottom left corner. In how many ways can it reach the upper left corner (marked **G**) in exactly **seven** moves?

5. *Note: For this question answers containing expressions such as $\frac{4\pi}{13}$ are acceptable.*

(a) The Jones family lives in a perfectly square house, 10m by 10m, which is placed exactly in the middle of a 40m by 40m section, entirely covered (except for the house) in grass. They keep the family pet, Dolly the sheep, tethered to the middle of one side of the house on a 15m rope. What is the area of the part of the lawn (in m^2) in which Dolly is able to graze? (See shaded area.)



The Jones | The Smiths

(b) The Jones' neighbours, the Smiths, have an identical section to the Jones but their house is located five metres to the North of the centre. Their pet sheep, Daisy, is tethered to the middle of the southern side of the house on a 20m rope. What is the area of the part of the lawn (in m^2) in which Daisy is able to graze?

Next we have part A of the final round of the BC senior mathematics competition. My thanks go to Jim Totten of the University College of the Cariboo, and Clint Lee of Okanagan University College for forwarding the material to me.

British Columbia Colleges
Senior High School Mathematics Contest, 2001
Final Round — Part A, Friday 4 May 2001

1. The number 2001 can be written as a difference of squares, $x^2 - y^2$, where x and y are positive integers, in four distinct ways. The sum of the four possible x values is:

(a) 55 (b) 56 (c) 879 (d) 1440 (e) 2880

2. Antonino goes to the local fruit stand and spends a total of \$20.01 on peaches and pears. If pears cost $18¢$ and peaches cost $33¢$, the maximum number of fruits Antonino could have bought is:

(a) 110 (b) 107 (c) 100 (d) 92 (e) 62

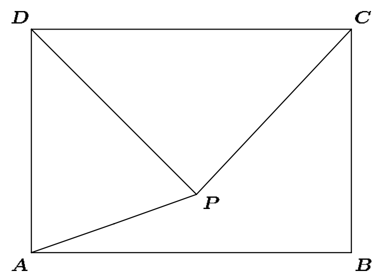
3. The value of $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$ is:

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{6}$ (e) 4

4. The point P is interior to the rectangle $ABCD$ such that $\overline{PA} = 3$ cm, $\overline{PC} = 5$ cm, and $\overline{PD} = 4$ cm. Then $\overline{PB}$, in centimetres, is:

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $3\sqrt{3}$

(d) $4\sqrt{2}$ (e) 2



5. Two overlapping spherical soap bubbles, whose centres are 50 mm apart, have radii of 40 mm and 30 mm. The two spheres intersect in a circle whose diameter, in millimetres, is:

(a) 36 (b) 48 (c) 50 (d) 54 (e) 64

6. The local theatre charges one dollar for the Sunday afternoon matinée. One Sunday the cashier finds that he has no change. Eight people arrive at the theatre; four have only a one-dollar coin (a loonie) and four have only a two-dollar coin (a toonie). Depending on how the people line up, the cashier may or may not be able to make change for every person in the line as they buy their tickets one at a time. Suppose that the eight people form a line in random order, without knowing who has a loonie and who has a toonie. Then the probability that the cashier will be able to make change for every person in the line is:

(a) $\frac{1}{70}$ (b) $\frac{1}{14}$ (c) $\frac{1}{7}$ (d) $\frac{1}{5}$ (e) $\frac{1}{4}$

7. There is a job opening at bcmath.com for a Webmaster. There are three required skills for the position: Writing, Design, and Programming. There are 45 applicants for the position. Of the 45 applicants, 80% have at least one of the required skills. Twenty of the applicants have at least design skills, 25 have at least writing skills, and 21 have at least programming skills. Twelve of the applicants have at least writing and design skills, fourteen have at least writing and programming skills, and eleven have at least design and programming skills. If only those applicants with all three skills will be interviewed, the number of applicants to be interviewed is:

(a) 3 (b) 7 (c) 8 (d) 9 (e) 11

8. Let $a \textcircled{L} b$ represent the operation on two numbers a and b, which selects the larger of the two numbers, with $a \textcircled{L} a = a$. Let $a \textcircled{S} b$ represent the operation which selects the smaller of the two numbers with $a \textcircled{S} a = a$. If a, b, and c are distinct numbers, and $a \textcircled{S} (b \textcircled{S} c) = (a \textcircled{S} b) \textcircled{L} (a \textcircled{S} c)$, then we must have:

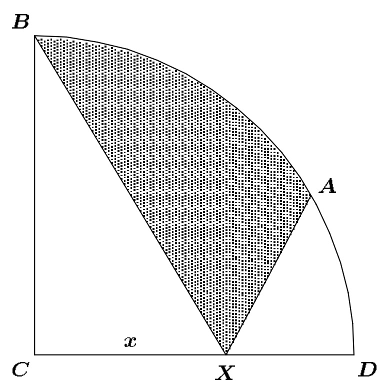
(a) $a < b$ and $a < c$ (b) $a > b$ and $a > c$ (c) $c < b < a$
(d) $c < a < b$ (e) $a < b < c$

9. The coordinates of the points A, B, and C are $(7, 4)$, $(3, 1)$, and $(0, k)$, respectively. The minimum value of $\overline{AC} + \overline{BC}$ is obtained when k equals:

(a) 1 (b) 1.7 (c) 1.9 (d) 2.5 (e) 4

10. Given the quarter circle BAD with radius $\overline{BC} = \overline{DC} = 1$, suppose that $\angle BCA = 60°$ and X is a point on segment DC with $\overline{CX} = x$. If the area of the shaded region BXA is one half the area of the quarter circle, then the value of x is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{\pi}{6}$
(d) $\frac{\pi}{4}$ (e) none of these



That completes the Skoliad Corner for this number. Send me your comments, suggestions, material and solutions for use in the corner.

Can you help?

One of our regular contributers, Dr. Eckard Specht <specht@iep352.nat.uni-magdeburg.de> has been looking for a very long time for two well-known books by Roger Arthur Johnson:

1. Advanced Euclidean Geometry, Dover Publications, Mineola, NY, 1960
2. Modern Geometry - An Elementary Treatise on the Geometry of the Triangle and the Circle, Houghton Mifflin, 1929.

Both are rare books and for a long time out of print. Attempts to order them by rare books shops (www.frugalfinder.com et al) on the internet failed. These books do not seem to exist in Germany — all libraries are missing these.

Does somebody have a copy of one or both that they would be willing to give or sell? If so, please contact Dr. Specht directly. Any help in obtaining a copy is appreciated. Thanks.