

## BOOK REVIEWS

ALAN LAW

*Geometry at Work*,  
 edited by Catherine A. Gorini,  
 published by the Mathematical Association of America (MAA Notes, No. 53),  
 2000, ISBN 0-88385-164-4, softcover, 300 pages, \$25.95 (U.S.).  
 Reviewed by **J. Chris Fisher**, *University of Regina, Regina, Saskatchewan.*

*Geometry at Work* is a collection of twenty essays in applied geometry, intended for “anyone having a college-level course in geometry.” The preface states that the articles can be used as “supplementary materials for teachers, resources for student projects, ideas for special lectures, inspiration for further research, or simply to broaden one’s awareness of geometry and its applications.” They are arranged into six sections:

Art and Architecture (4 papers, 34 pages),

Vedic Civilization (2 papers, 17 pages),

The Classroom (teaching descriptive geometry and ethnomathematics; 2 papers, 10 pages),

Engineering (robotics, structural engineering, Geographical Information Systems, and medical imaging; 5 papers, 47 pages),

Decision-Making Processes (voting and computer learning; 2 papers, 25 pages),

Mathematics and Science (number theory, optimization, graph theory, quantum mechanics, and crystallography; 5 papers, 69 pages).

The level is that of the various journals of the *Mathematical Association of America*, although the mathematical content is variable — there were several articles with insufficient content for publication in any mathematical journal, and one whose level was well beyond the intended audience. This exception is “Three-Dimensional Topology and Quantum Physics” by Louis H. Kauffman. It is a marvelous survey of quantum mechanics from the beginning in 1924 with DeBroglie’s “fantastic notion” that inspired Schrodinger’s equations, through the recent advances in knot theory. Although one frequently sees hints that the pieces are connected, I have never before seen the explanation of how the pieces fit together. The story is fascinating, informative, very well motivated, and told in just 11 pages. The geometry involved is the intuitive notion of topology behind Reidemeister’s basic ideas about knots, but there are many advanced concepts — eigenvalues of Hilbert-space operators, for one — that are an essential part of the development.

Before continuing my review, here are a few very personal opinions. The motivation behind collecting applications seems based on the notion that geometry is a collection of usable facts. I suppose so, to some extent, but this point of view unfortunately reinforces the beliefs of those responsible for cutting back the teaching of geometry in high schools across North America. In many schools geometry is taught as a bunch of facts that can be stuck here and there as topics in a general mathematics course. However, geometry used to be, and always should be, the first place where students get to see deductive reasoning. The intuitive nature of the subject helps a person develop a feeling for the role of proof in mathematics and science. The abstract axiom systems studied in university courses do not serve as an effective pedagogical tool. For a full discussion of this “application” of geometry I recommend the article by Jim McClure, “Start Where They Are: Geometry as an Introduction to Proof,” *Amer. Math. Monthly* **107**:1 (January 2000) 44–52.

The articles are all well written. Basic geometry is represented in several articles in its historic role of guiding measurements both in traditional settings and in medical imaging.—Somewhat more advanced geometry is needed to analyze aperiodic tilings, where symmetry is replaced by statistical symmetry. Convexity plays the key role in several articles, of which two involve optimization, one involves number theory (applying Minkowski's geometry of numbers to prove three familiar results), and one deals with robotics: how many fingers are needed to “grasp” a rigid 3-dimensional object? (Answer: 7 or so, depending on what is meant by *grasp*.)

The real question: is it worthwhile to purchase the book? My opinion is neutral here. The answer depends on how much money and shelf space is available to you or to your library. Eleven of the essays are of questionable value: either they lack mathematical content, or similar articles are readily available elsewhere, or they are too sketchy. (One of these sketchy articles looked promising, but its three references were all in Russian.) That leaves nine articles that I found worth reading carefully. Any reader of ***CRUX with MAYHEM*** would likewise find several interesting articles in this collection. Had somebody asked my opinion, I would have suggested that these articles appear in a widely accessible refereed journal. Far more valuable than this monograph would have been a bibliography that provides a brief description of articles involving applications and classifies them according to the mathematics required and the level of the exposition.