PROBLEMS

Problem proposals and solutions should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. A1C 5S7. Proposals should be accompanied by a solution, together with references and other insights which are likely to be of help to the editor. When a submission is submitted without a solution, the proposer must include sufficient information on why a solution is likely. An asterisk ($\star$) after a number indicates that a problem was submitted without a solution.

In particular, original problems are solicited. However, other interesting problems may also be acceptable provided that they are not too well known, and references are given as to their provenance. Ordinarily, if the originator of a problem can be located, it should not be submitted without the originator's permission.

To facilitate their consideration, please send your proposals and solutions on signed and separate standard $8\frac{1}{2}$"×11" or A4 sheets of paper. These may be typewritten or neatly hand-written, and should be mailed to the Editor-in-Chief, to arrive no later than **1 March 2001**. *They may also be sent by email to* **crux-editors@cms.math.ca**. *(It would be appreciated if email proposals and solutions were written in LaTeX). Graphics files should be in epic format, or encapsulated postscript. Solutions received after the above date will also be considered if there is sufficient time before the date of publication. Please note that we do not accept submissions sent by FAX.*

2551. *Proposed by Panos E. Tsaoussoglou, Athens, Greece.*

Suppose that a_k $(1 \le k \le n)$ are positive real numbers. Let $e_{j,k} = (n-1)$ if $j = k$ and $e_{j,k} = (n-2)$ otherwise. Let $d_{j,k} = 0$ if $j = k$ and $d_{j,k} = 1$ otherwise.

Prove that

$$\prod_{j=1}^{n} \sum_{k=1}^{n} e_{j,k} a_k^2 \ \ge\ \prod_{j=1}^{n} \left(\sum_{k=1}^{n} d_{j,k} a_k \right)^2 .$$

2552. *Proposed by Aram Tangboondouangjit, Carnegie Mellon University, Pittsburgh, PA, USA.*

Suppose that a, b, $c > 0$. If $x \ge \frac{a+b+c}{3\sqrt{3}} - 1$, prove that

$$\frac{(b+cx)^2}{a} + \frac{(c+ax)^2}{b} + \frac{(a+bx)^2}{c} \ \ge\ abc\,.$$

2553. *Proposed by Aram Tangboondouangjit, Carnegie Mellon University, Pittsburgh, PA, USA.*

Find all real roots of the equation

$$\frac{\left(\sqrt{2x^2-2x+12} - \sqrt{x^2-5}\right)^3}{(5x^2-2x-3)\sqrt{2x^2-2x+12}} \ =\ \frac{2}{9}\,.$$

2554. *Proposed by Aram Tangboondouangjit, Carnegie Mellon University, Pittsburgh, PA, USA.*

In triangle ABC, prove that at least one of the quantities

$$(a+b-c)\tan^2\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right),$$
$$(-a+b+c)\tan^2\left(\frac{B}{2}\right)\tan\left(\frac{C}{2}\right),$$
$$(a-b+c)\tan^2\left(\frac{C}{2}\right)\tan\left(\frac{A}{2}\right),$$

is greater than or equal to $\frac{2r}{3}$, where r is the radius of the incircle of $\triangle ABC$.

2555. *Proposed by Aram Tangboondouangjit, Carnegie Mellon University, Pittsburgh, PA, USA.*

In any triangle ABC, show that

$$\sum_{\text{cyclic}} \frac{1}{\tan^3\frac{A}{2} + \left(\tan\frac{B}{2} + \tan\frac{C}{2}\right)^3} < \frac{4\sqrt{3}}{3}.$$

2556*. *Proposed by Mohammed Aassila, CRM, Université de Montréal, Montréal, Québec.*

A lattice point is called *visible* (from the origin) if its coordinates are coprime numbers. Is there any lattice point whose distance from each visible lattice point is at least 2000?

2557. *Proposed by Gord Sinnamon, University of Western Ontario, London, Ontario, and Hans Heinig, McMaster University, Hamilton, Ontario.*

(a) Show that for all positive sequences $\{x_i\}$ and all integers $n > 0$,

$$\sum_{k=1}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j} x_i \le 2\sum_{k=1}^{n}\left(\sum_{j=1}^{k} x_j\right)^2 x_k^{-1}.$$

(b)* Does the above inequality remain true without the factor 2?
(c)* [Proposed by the editors] What is the minimum constant c that can replace the factor 2 in the above inequality?

2558. *Proposed by Peter Y. Woo, Biola University, La Mirada, CA, USA.*

Let Z be a half-plane bounded by a line L. Let A, B and C be any three points on L such that C lies between A and B. Denote the three semicircles in Z on AB, AC and CB as diameters by K_0, K_1 and K_2 respectively. Let F be the family of semicircles in Z with diameters on L (including all half-lines in Z perpendicular to L). Denote by f_{XY} the unique semicircle passing through the pair of distinct points X, Y in $Z \cup L$. Let P, Q, R, be three points on K_2, K_1, K_0, respectively.

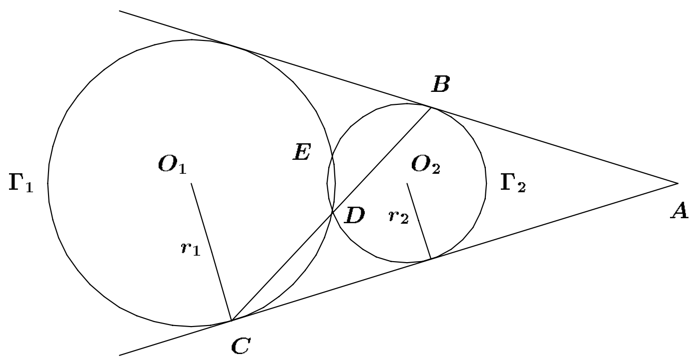
If f_{AP}, f_{BQ} and f_{CR} concur at T, and the lines AP, BQ, CR concur at S, prove that f_{AP}, f_{BQ} and f_{CR} are orthogonal to K_2, K_1, K_0, respectively, and that the circle PQR is tangent to each semicircle K_j, ($j = 0, 1, 2$).

2559 *Proposed by Ho-joo Lee, student, Kwangwoon University, Seoul, South Korea.*

Triangle ABC has incentre I. Show that $CA + AI = CB$ if and only if $\angle CAB = 2\angle ABC$.

2560★ *Proposed by Václav Konečný, Ferris State University, Big Rapids, MI, USA.*

Lines AB and AC are common tangents to the circles Γ_1 and Γ_2 with distinct radii r_1 and r_2 respectively, as shown.



B is a point of tangency on Γ_2 and C is a point of tangency on Γ_1. The intersection points of the circles, D and E, exist, CDB is a straight line, and $CD = DB$.

Construct such a figure using straightedge and compass.

2561. *Proposed by Hassan A. ShahAli, Tehran, Iran.*

Let M disks from N different colours be placed in a row such that k_i disks are from the i^{th} colour ($i = 1, 2, \ldots, N$) and $k_1 + k_2 + \cdots + k_N = M$.

A move is an exchange of two adjacent disks.

Determine the smallest number of moves needed to rearrange the row such that all disks of the same colour are adjacent to one another.

2562. *Proposed by Bernardo Recamán Santos, Colegio Hacienda Los Alcaparros, Bogotá, Colombia.*

(a) Show that for all sufficiently large n, it is possible to find a set of n (not necessarily distinct) positive integers whose sum is the square root of their product.

(b)★ Are there infinitely many n for which there is a unique set of n numbers with property (a)?