# PROBLEMS

*Problem proposals and solutions should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. A1C 5S7. Proposals should be accompanied by a solution, together with references and other insights which are likely to be of help to the editor. When a submission is submitted without a solution, the proposer must include sufficient information on why a solution is likely. An asterisk* ($\star$) *after a number indicates that a problem was submitted without a solution.*

*In particular, original problems are solicited. However, other interesting problems may also be acceptable provided that they are not too well known, and references are given as to their provenance. Ordinarily, if the originator of a problem can be located, it should not be submitted without the originator's permission.*

*To facilitate their consideration, please send your proposals and solutions on signed and separate standard* $8\frac{1}{2}''\times 11''$ *or A4 sheets of paper. These may be typewritten or neatly hand-written, and should be mailed to the Editor-in-Chief, to arrive no later than* **1 April 1999**. *They may also be sent by email to* **crux-editors@cms.math.ca**. (*It would be appreciated if email proposals and solutions were written in* LaTeX). *Graphics files should be in epic format, or encapsulated postscript. Solutions received after the above date will also be considered if there is sufficient time before the date of publication. Please note that we do not accept submissions sent by FAX.*

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**2374**. [1998: 365] **(Correction)** *Proposed by Toshio Seimiya, Kawasaki, Japan.*

Given triangle $ABC$ with $\angle BAC > 60^\circ$. Let $M$ be the mid-point of $BC$. Let $P$ be any point in the plane of $\triangle ABC$.

Prove that $AP + BP + CP \geq 2AM$.

**2376**. *Proposed by Albert White, St. Bonaventure University, St. Bonaventure, NY, USA.*

Suppose that $ABC$ is a right-angled triangle with the right angle at $C$. Let $D$ be a point on hypotenuse $AB$, and let $M$ be the mid-point of $CD$. Suppose that $\angle AMD = \angle BMD$. Prove that

1. $\overline{AC}^2\,\overline{MC}^2 + 4[ABC]\,[BCD] \;=\; \overline{AC}^2\,\overline{MB}^2$;
2. $4\overline{AC}^2\,\overline{MC}^2 - \overline{AC}^2\overline{BD}^2 \;=\; 4[ACD]^2 - 4[BCD]^2$,

where $[XYZ]$ denotes the area of $\triangle XYZ$.

(This is a continuation of problem **1812**, [1993: 48].)

**2377**. *Proposed by Nikolaos Dergiades, Thessaloniki, Greece.*
Let $ABC$ be a triangle and $P$ a point inside it. Let $BC = a$, $CA = b$, $AB = c$, $PA = x$, $PB = y$, $PC = z$, $\angle BPC = \alpha$, $\angle CPA = \beta$ and $\angle APB = \gamma$.

Prove that $ax = by = cz$ if and only if $\alpha - A = \beta - B = \gamma - C = \frac{\pi}{3}$.

**2378**. *Proposed by David Doster, Choate Rosemary Hall, Wallingford, Connecticut, USA.*
Find the exact value of: $\cot\left(\dfrac{\pi}{22}\right) - 4\cos\left(\dfrac{3\pi}{22}\right)$.

**2379**. *Proposed by D.J. Smeenk, Zaltbommel, the Netherlands.*
Suppose that $M_1$, $M_2$ and $M_3$ are the mid-points of the altitudes from $A$ to $BC$, from $B$ to $CA$ and from $C$ to $AB$ in $\triangle ABC$. Suppose that $T_1$, $T_2$ and $T_3$ are the points where the excircles to $\triangle ABC$ opposite $A$, $B$ and $C$, touch $BC$, $CA$ and $AB$.

Prove that $M_1T_1$, $M_2T_2$ and $M_3T_3$ are concurrent.

Determine the point of concurrency.

**2380**. *Proposed by Bill Sands, University of Calgary, Calgary, Alberta.*
When the price of a certain book in a store is reduced by 1/3 and rounded to the nearest cent, the cents and dollars are switched. For example, if the original price was \$43.21, the new price would be \$21.43 (this does not satisfy the "reduced by 1/3" condition, of course). What was the original price of the book? [For the benefit of readers unfamiliar with North American currency, there are 100 cents in one dollar.]

**2381**. *Proposed by Angel Dorito, Geld, Ontario.*
Solve the equation $\log_2 x = \log_4(x+1)$.

**2382**. *Proposed by Mohammed Aassila, Université Louis Pasteur, Strasbourg, France.*
If $\triangle ABC$ has inradius $r$ and circumradius $R$, show that

$$\cos^2\left(\frac{B-C}{2}\right) \geq \frac{2r}{R}.$$

**2383**. *Proposed by Mohammed Aassila, Université Louis Pasteur, Strasbourg, France.*
Suppose that three circles, each of radius 1, pass through the same point in the plane. Let $A$ be the set of points which lie inside at least two of the circles. What is the least area that $A$ can have?

**2384**. *Proposed by Paul Bracken, CRM, Université de Montréal, Québec.*
Prove that $2(3n-1)^n \geq (3n+1)^n$ for all $n \in \mathbb{N}$.

**2385**. *Proposed by Joaquín Gómez Rey, IES Luis Buñuel, Alcorcón, Madrid, Spain.*

A die is thrown $n \geq 3$ consecutive times. Find the probability that the sum of its $n$ outcomes is greater than or equal to $n + 6$ and less than or equal to $6n - 6$.

**2386**$^\star$. *Proposed by Clark Kimberling, University of Evansville, Evansville, IN, USA.*

Write

$$\begin{matrix} 1 \\ \phantom{1} \end{matrix} \to \begin{matrix} 1 \\ 1 \end{matrix} \to \begin{matrix} 3 \\ 1 \end{matrix} \to \begin{matrix} 4 & 1 \\ 1 & 3 \end{matrix} \to \begin{matrix} 6 & 2 & 1 \\ 1 & 3 & 4 \end{matrix} \to \begin{matrix} 8 & 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 & 6 \end{matrix} \to$$

(The last ten numbers shown indicate that up to this point, eight $1$'s, one $2$, three $3$'s, two $4$'s and one $6$ have been written.)

(a) If this is continued indefinitely, will $5$ eventually appear?

(b) Will every positive integer eventually be written?

Note: $11$ is a number and not two $1$'s.

**2387**. *Proposed by Walther Janous, Ursulinengymnasium, Innsbruck, Austria.*

For fixed $p \in \mathbb{N}$, consider the power sums

$$S_p(n) := \sum_{k=1}^{n} (2k-1)^p, \qquad \text{where } n \geq 1,$$

so that $S_p(n)$ is a polynomial in $n$ of degree $p + 1$ with rational coefficients. Prove that

(a) If all coefficients of $S_p(n)$ are integers, then $p = 2^m - 1$ for some $m \in \mathbb{N}$.

(b)$^\star$ The only values of $p$ yielding such polynomials are $p = 1$ and $p = 3$ (with $S_1(n) = n^2$ and $S_3(n) = 2n^4 - n^2$).