We warmly welcome proposals for problems and solutions. With the new schedule of eight issues per year, we request that solutions be submitted by 1 June 1997, for publication in the issue 5 months ahead; that is, issue 6. We also request that **only students** submit solutions (see editorial), but we will consider particularly elegant or insightful solutions for others. Since this rule is only being implemented now, you will see solutions from many people in the next few months, as we clear out the old problems from Mayhem.

High School Problems

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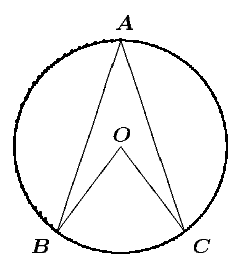
H217. Let a_1, a_2, a_3, a_4, a_5 be a five-term geometric sequence satisfying the inequality $0 < a_1 < a_2 < a_3 < a_4 < a_5 < 100$, where each term is an integer. How many of these five-term geometric sequences are there? (For example, the sequence 3, 6, 12, 24, 48 is a sequence of this type).

H218. A Star Trek logo is inscribed inside a circle with centre O and radius 1, as shown. Points A, B, and C are selected on the circle so that $AB = AC$ and arc BC is minor (that is $ABOC$ is not a convex quadrilateral). The area of figure $ABOC$ is equal to $\sin m^\circ$, where $0 < m < 90$ and m is an integer. Furthermore, the length of arc AB (shaded as shown) is equal to $a\pi/b$, where a and b are relatively prime integers. Let $p = a + b + m$.



(i) If $p = 360$, and m is composite, determine all possible values for m.

(ii) If m and p are both prime, determine the value of p.

H219. Consider the infinite sum

$$S = \frac{a_0}{10^0} + \frac{a_1}{10^2} + \frac{a_2}{10^4} + \frac{a_3}{10^6} + \cdots ,$$

where the sequence $\{a_n\}$ is defined by $a_0 = a_1 = 1$, and the recurrence relation $a_n = 20a_{n-1} + 12a_{n-2}$ for all positive integers $n \geq 2$. If $\sqrt{S}$ can be expressed in the form $\frac{a}{\sqrt{b}}$, where a and b are relatively prime positive integers, determine the ordered pair (a, b).

H220. Let S be the sum of the elements of the set

$$\{1, 2, 3, \dots , (2p)^n - 1\}.$$

Let T be the sum of the elements of this set whose representation in base $2p$ consists only of digits from 0 to $p - 1$.

Prove that $2n \times \frac{T}{S} = (p - 1)/(2p - 1)$.

Advanced Problems

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A193. If $f(x, y)$ is a convex function in x for each fixed y, and a convex function in y for each fixed x, is $f(x, y)$ necessarily a convex function in x and y?

A194. Let H be the orthocentre (point where the altitudes meet) of a triangle ABC. Show that if $AH : BH : CH = BC : CA : AB$, then the triangle is equilateral.

A195. Compute $\tan 20° \tan 40° \tan 60° \tan 80°$.

A196. Show that $r^2 + r_a^2 + r_b^2 + r_c^2 \geq 4K$, where r, r_a, r_b, r_c, and K are the inradius, exradii, and area respectively of a triangle ABC.

Challenge Board Problems

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C70. Prove that the group of automorphisms of the dodecahedron is S_5, the symmetric group on five letters, and that the rotation group of the dodecahedron (the subgroup of automorphisms preserving orientation) is A_5.

C71. Let L_1, L_2, L_3, L_4 be four general lines in the plane. Let p_{ij} be the intersection of lines L_i and L_j. Prove that the circumcircles of the four triangles $p_{12}p_{23}p_{31}$, $p_{23}p_{34}p_{42}$, $p_{34}p_{41}p_{13}$, $p_{41}p_{12}p_{24}$ are concurrent.

C72. A finite group G acts on a finite set X transitively. (In other words, for any $x, y \in X$, there is a $g \in G$ with $g \cdot x = y$.) Prove that there is an element of G whose action on X has no fixed points.