

# MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as a **Mathematical Journal for and by High School and University Students**. It continues, with the same emphasis, as an integral part of *Crux Mathematicorum with Mathematical Mayhem*.

All material intended for inclusion in this section should be sent to the Mayhem Editor, Naoki Sato, Department of Mathematics, University of Toronto, Toronto, ON Canada M5S 1A1. The electronic address is

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The Assistant Mayhem Editor is Cyrus Hsia (University of Toronto). The rest of the staff consists of Richard Hoshino (University of Waterloo), Wai Ling Yee (University of Waterloo), and Adrian Chan (Upper Canada College).

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## Editorial

It gives me great pleasure to unveil the premiere issue of “*Crux Mathematicorum with Mathematical Mayhem*”. This merger has been in the works for quite some time, and it has finally been successfully realized.

For the benefit of those who have not heard of **Mayhem**, I will provide a brief description. **Mayhem** was founded in 1988 by two high school students Ravi Vakil and Patrick Surry, who wished to establish a journal specifically oriented towards students, and totally operated by students. Although the journal has been passed down through many hands, and though it has not always been easy, this mandate has always been resolutely upheld; it has made **Mayhem** a unique and exceptional journal. And rest assured, we will still be running our share of “*Crux with Mayhem*”.

Our features include articles, olympiads, and a problems section. The material is generally focused towards contests and olympiads, and how to prepare for them, and the topics range from high school mathematics to undergraduate material. We cannot emphasize enough that we are a journal dedicated to mathematics students. I myself am a fourth-year student at the University of Toronto, and Cyrus Hsia (the **Mayhem** Assistant Editor) is a third-year student, also at the University of Toronto.

Our fearless staff also consists of undergraduate and high school students.

This brings me to my next point. After considerable discussion, “**Mayhem**” has decided to restrict itself to publishing solutions only from students. The rationale behind this move is that the **Crux** problems already draw many solutions, and if the same people were to respond to our problems, which are considerably easier, it would simply overwhelm the section. We know that

there are many non-students who have contributed to the problems sections over the years, who have our full gratitude, and hope they understand our position. We are, however, prepared to make exceptions in, well, exceptional cases.

However, we warmly welcome submissions for articles and problems from all people. Back issues are available; the information is inside the back cover. Any correspondence about **Mayhem** should be sent to **Mathematical Mayhem**, c/o Naoki Sato, Department of Mathematics, University of Toronto, M5S 1A1, or at the e-mail address <mayhem@math.toronto.edu>. Subscriptions, however, should be sent to the Canadian Mathematical Society offices, as mentioned on the inside back cover.

Well, I think that's about it. For people who have subscribed to **Mayhem**, welcome back, I know it's been a long wait. I look forward to working with Bruce Shawyer on our new project (I think he will bring a certain "discipline" to **Mayhem**, but we will resist it as much as possible. Don't tell him I said that.) Here's to a new year and a new era for **Mayhem**.

Naoki Sato  
Mayhem Editor

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## Shreds and Slices

### Positive Matrices and Positive Eigenvalues

**Theorem.** An  $n \times n$  matrix  $M$  with positive real entries has at least one positive eigenvalue.

**Proof.** Let  $S = \{(x_1, x_2, \dots, x_n) \mid x_1, x_2, \dots, x_n \geq 0, x_1^2 + x_2^2 + \dots + x_n^2 = 1\}$ ; that is,  $S$  is the portion of the unit sphere in  $\mathbb{R}^n$  with all coordinates non-negative.

Define the map  $f : S \rightarrow S$  by  $f(\vec{v}) = \frac{M\vec{v}}{|M\vec{v}|}$ . Since  $M$  has all positive real entries, for any  $\vec{v} \in S$ ,  $M\vec{v}$  also has all non-negative coordinates, so  $f$  is well-defined, and does indeed map  $S$  into  $S$ .

Note that  $S$  is a closed, simply connected set. Then by Brouwer's Fixed Point Theorem, there is a fixed point of  $f$ ; that is, for some  $\vec{v} \in S$ ,  $f(\vec{v}) = \vec{v} = \frac{M\vec{v}}{|M\vec{v}|} \Rightarrow M\vec{v} = \lambda\vec{v}$ , for some positive value  $\lambda$  (since  $\lambda$  cannot be zero), namely  $\lambda = |M\vec{v}|$ . This  $\lambda$  is a positive eigenvalue of  $M$ .

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## Newton's Relations

Given  $n$  reals  $a_1, a_2, \dots, a_n$ , let  $S_k$  be the sum of the products of the  $a_i$  taken  $k$  at a time, and let  $P_k = a_1^k + a_2^k + \dots + a_n^k$ . Consider the generating functions

$$\begin{aligned} S_0 + S_1x + S_2x^2 + \dots + S_nx^n \\ &= 1 + (a_1 + a_2 + \dots + a_n)x + (a_1a_2 + a_1a_3 + \dots + a_{n-1}a_n)x^2 \\ &\quad + \dots + a_1a_2 \dots a_nx^n \\ &= (1 + a_1x)(1 + a_2x) \dots (1 + a_nx) \end{aligned}$$

and

$$\begin{aligned} P_0 - P_1x + P_2x^2 - \dots \\ &= (1 - a_1x + a_1^2x^2 - \dots) + (1 - a_2x + a_2^2x^2 - \dots) \\ &\quad + \dots + (1 - a_nx + a_n^2x^2 - \dots) \\ &= \frac{1}{1 + a_1x} + \frac{1}{1 + a_2x} + \dots + \frac{1}{1 + a_nx}. \end{aligned}$$

Their product is

$$\begin{aligned} (n - P_1x + P_2x^2 - \dots)(1 + S_1x + S_2x^2 + \dots) \\ &= (1 + a_1x)(1 + a_2x) \dots (1 + a_nx) \\ &\quad \times \left( \frac{1}{1 + a_1x} + \frac{1}{1 + a_2x} + \dots + \frac{1}{1 + a_nx} \right), \end{aligned}$$

since  $P_0 = n$  and  $S_0 = 1$ .

We claim the expression is equal to  $n + (n-1)S_1x + (n-2)S_2x^2 + \dots + S_{n-1}x^{n-1}$ . To see this, consider the coefficient of  $x^k$ . Since the expression is symmetric, the coefficient is some multiple of  $S_k$ . How many times does the term  $a_1a_2 \dots a_kx^k$  appear? It must have appeared in the product  $(1 + a_1x)(1 + a_2x) \dots (1 + a_nx)$  as  $a_1a_2 \dots a_ka_l$ , where  $k < l \leq n$ , before having the term  $a_l$  divided out. There are  $n - k$  choices for  $l$ , and hence the coefficient is  $(n - k)S_k$ .

Hence,

$$\begin{aligned} (n - P_1x + P_2x^2 - \dots)(1 + S_1x + S_2x^2 + \dots) \\ &= n + (n-1)S_1x + (n-2)S_2x^2 + \dots + S_{n-1}x^{n-1}, \end{aligned}$$

and equating coefficients:

$$\begin{aligned} nS_1 - P_1 &= (n-1)S_1 \\ nS_2 - S_1P_1 + P_2 &= (n-2)S_2 \\ nS_3 - S_2P_1 + S_1P_2 - P_3 &= (n-3)S_3 \\ &\dots \\ nS_{n-1} - S_{n-2}P_1 + S_{n-3}P_2 - \dots + (-1)^{n-1}P_{n-1} &= S_{n-1}, \end{aligned}$$

or

$$\begin{aligned}
 P_1 - S_1 &= 0 \\
 P_2 - S_1 P_1 + 2S_2 &= 0 \\
 P_3 - S_1 P_2 + S_2 P_1 - 3S_3 &= 0 \\
 &\dots \\
 P_{n-1} - S_1 P_{n-2} + S_2 P_{n-3} - \dots + (-1)^{n-1} (n-1) S_{n-1} &= 0, \\
 &\text{and} \\
 P_m - S_1 P_{m-1} + S_2 P_{m-2} - \dots + (-1)^n P_{m-n} S_n &= 0 \\
 &\text{for } m \geq n.
 \end{aligned}$$

The last equation is the well-known recursion sequence for the  $P_i$ , and the previous equations (known as Newton's relations) can help pin down the values of  $P_1, P_2, \dots, P_{n-1}$ , or vice-versa.

**Problem.** If

$$\begin{aligned}
 x + y + z &= 1, \\
 x^2 + y^2 + z^2 &= 2, \\
 x^3 + y^3 + z^3 &= 3,
 \end{aligned}$$

determine the value of  $x^4 + y^4 + z^4$ .

**Solution.** Newton's relations become

$$\begin{aligned}
 P_1 - S_1 &= 1 - S_1 = 0, \\
 P_2 - S_1 P_1 + 2S_2 &= 2 - S_1 + 2S_2 = 0, \\
 P_3 - S_1 P_2 + S_2 P_1 - 3S_3 &= 3 - 2S_1 + S_2 - 3S_3 = 0,
 \end{aligned}$$

which imply that  $S_1 = 1$ ,  $S_2 = -1/2$ , and  $S_3 = 1/6$ . Also,  $P_4 - 3S_1 + 2S_2 - S_3 = 0 \Rightarrow P_4 = 25/6$ .

Here is the last problem of the 1995 Japan Mathematical Olympiad, Final Round.

**Problem.** Let  $1 \leq k \leq n$  be positive integers. Let  $a_1, a_2, \dots, a_k$  be complex numbers satisfying

$$\begin{aligned}
 a_1 + a_2 + \dots + a_k &= n \\
 a_1^2 + a_2^2 + \dots + a_k^2 &= n \\
 &\dots \\
 a_1^k + a_2^k + \dots + a_k^k &= n
 \end{aligned}$$

Show that  $(x + a_1)(x + a_2) \cdots (x + a_n) = x^k + \binom{n}{1} x^{k-1} + \binom{n}{2} x^{k-2} + \dots + \binom{n}{k}$ .

**Solution.** Given  $P_1 = P_2 = \dots = P_k = n$ , we must find  $S_1, S_2, \dots, S_k$ . We will prove that  $S_m = \binom{n}{m}$  by induction. Clearly  $S_1 = P_1 = n = \binom{n}{1}$ .

Now, for some  $m$ , assume  $S_1 = \binom{n}{1}$ ,  $S_2 = \binom{n}{2}$ ,  $\dots$ ,  $S_{m-1} = \binom{n}{m-1}$ . Then by the equations above,  $P_m - S_1 P_{m-1} + S_2 P_{m-2} - \dots + (-1)^{m-1} S_{m-1} P_1 + m(-1)^m S_m = 0$ , or

$$n - n \binom{n}{1} + n \binom{n}{2} - \dots + (-1)^{m-1} n \binom{n}{m-1} + m(-1)^m S_m = 0$$

so that

$$\frac{m}{n} S_m = \binom{n}{m-1} - \binom{n}{m-2} + \binom{n}{m-3} - \dots = \binom{n-1}{m-1},$$

and further,

$$S_m = \frac{n}{m} \binom{n-1}{m-1} = \binom{n}{m}.$$

So by induction, we are done.

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## Mathematically Correct Sayings

[The following shred/slice appeared in the newsgroup `rec.humor.funny`.]

After applying some simple algebra to some trite phrases and cliches, a new understanding can be reached of the secret to wealth and success. Here it goes.

Knowledge is Power,  
Time is Money,  
and as everyone knows, Power is Work divided by Time.

So, substituting algebraic equations for these time worn bits of wisdom, we get:

$$K = P \tag{1}$$

$$T = M \tag{2}$$

$$P = W/T \tag{3}$$

Now, do a few simple substitutions. Put  $W/T$  in for  $P$  in equation (1), which yields:

$$K = W/T \tag{4}$$

Put  $M$  in for  $T$  into equation (4), which yields:

$$K = W/M \tag{5}$$

Now we've got something. Expanding back into English, we get: Knowledge equals Work divided by Money.

What this MEANS is that:

1. The More You Know, the More Work You Do, and
2. The More You Know, the Less Money You Make.

Solving for Money, we get:

$$M = W/K \quad (6)$$

Money equals Work divided by Knowledge.

From equation (6) we see that Money approaches infinity as Knowledge approaches 0, regardless of the Work done.

What THIS MEANS is: The More you Make, the Less you Know.

Solving for Work, we get

$$W = M \times K \quad (7)$$

Work equals Money times Knowledge

From equation (7) we see that Work approaches 0 as Knowledge approaches 0.

What THIS MEANS is: The stupid rich do little or no work.

Working out the socioeconomic implications of this breakthrough is left as an exercise for the reader.

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## Contest Dates

Here are some upcoming (or in some cases, already past) contest dates to mark on your calendar.

Contest	Grade	Date
Gauss	Grades 7 & 8	Wednesday, May 14, 1997
Pascal	Grade 9	Wednesday, February 19, 1997
Cayley	Grade 10	Wednesday, February 19, 1997
Fermat	Grade 11	Wednesday, February 19, 1997
Euclid	Grade 12	Tuesday, April 15, 1997
Descartes	Grades 12 & 13	Wednesday, April 16, 1997
CIMC	Grades 10 & 11	Wednesday, April 16, 1997
AJHSME	Grades 7 & 8	Thursday, November 21, 1996
AHSME	High School	Thursday, February 13, 1997
AIME	High School	Thursday, March 20, 1997
USAMO	High School	Thursday, May 1, 1997
AJHSME	Grades 7 & 8	Thursday, November 20, 1997
COMC	High School	Wednesday, November 27, 1996
CMO	High School	Wednesday, March 26, 1997
APMO	High School	March, 1997
IMO	High School	July 18 – 31, 1997

## A Journey to the Pole — Part I

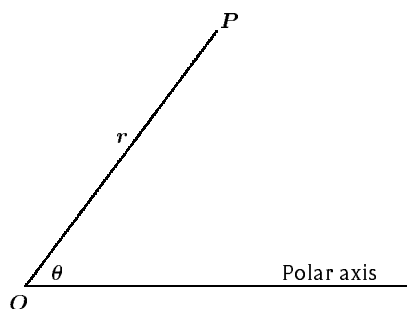
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student, Universidad Complutense de Madrid  
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For those of us who can not seem to get a strong grip on synthetic geometry, analytic geometry comes in handy. Even though polar coordinates can be superior to rectangular coordinates in some situations, they are systematically ignored by instructors and students alike. The purpose of this series is to introduce their uses with the idea that, as is always happening in mathematics, with a little ingenuity, the concepts central to polar coordinates can be applied elsewhere. This first article uses polar coordinates in elementary geometry.

### Definition

In polar coordinates, the position of a point  $P$  is determined by the distance  $r$  from a point  $O$  called the *pole* and the angle  $\theta$  between  $OP$  and a semi-infinite line called the *polar axis*. By convention, the polar axis is taken



to be the positive  $x$ -axis, and the transformation from polar to cartesian coordinates is given by  $x = r \cos \theta$ ,  $y = r \sin \theta$ . The inverse change of coordinates is not so straightforward; The obvious expressions are  $r = \sqrt{x^2 + y^2}$ ,  $\theta = \arctan(\frac{y}{x})$ , but the equation for  $\theta$  is not single-valued, even if  $\theta$  is restricted to  $[0, 2\pi)$ , and  $\theta$  is undefined at the origin. Fortunately, we need not worry about this: when

handling curves in polar coordinates, the change from rectangular to polar coordinates is of little use, and it is convenient to allow  $r$  and  $\theta$  to take on all real values. With this provision, a point can be referred to by an infinite set of coordinate pairs:  $(r, \theta) = ((-1)^n r, \theta + n\pi)$ . Unless you want to do multiple integrals, this is not a problem, but rather something to exploit!

### Polar Curves

Polar curves are usually written in the form  $r = r(\theta)$ , and unlike curves of the form  $y = y(x)$ , they can be closed and need not be simple (they can intersect themselves). Implicit curves of the form  $f(r, \theta) = 0$  can be even more general. From a cartesian equation, the substitution  $x = r \cos \theta$ ,  $y = r \sin \theta$  yields a polar expression. Throughout this article, I have tried to avoid this whenever possible, and it turns out that it is always possible.

Some symmetries of the curves can be detected by checking the functions  $r(\theta)$  or  $f(r, \theta)$  above for the following simple properties (this is not a complete list):

- The curve is symmetric about the pole if  $r(\theta) = r(\theta + \pi)$  or  $f(-r, \theta) = f(r, \theta)$
- The curve has  $n$ -fold symmetry about the pole if  $r(\theta) = r(\theta + \frac{2\pi}{n})$
- The curve is symmetric about the polar axis if  $r(\theta) = r(-\theta)$
- The curve is symmetric about a line at an angle  $\phi$  to the polar axis if  $r(\theta) = r(2\phi - \theta)$

The following transformations are also useful:

- Any curve can be rotated through  $\phi$  by substituting  $\theta - \phi$  for  $\theta$
- The  $x$ - and  $y$ -axes can be permuted by substituting  $\frac{\pi}{2} - \theta$  for  $\theta$

**Example 1.** The equation of a circle of radius  $a$  centered at the origin is  $r = a$ .

**Example 2.** The equation of a line passing through the origin at an angle  $\phi$  to the polar axis is  $\theta = \phi$ .

**Exercise 1.** Find the equation of a line at an angle  $\phi$  to the polar axis passing at a distance  $d$  to the pole.

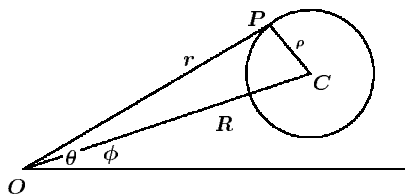
**Exercise 2.** Identify the curve  $r = 2a \cos \theta$ .

### The Cosine Law

More often than not, when working in polar coordinates, one uses nothing but trigonometry, and the cosine law is the starting point of many derivations. If you think about it, it comes closest to being a 'vector addition rule' to use if you need to translate a curve, although this is best done in rectangular coordinates. I will not give a translation rule, because it is cumbersome and is of little use. Instead, I will use the cosine law to derive the equation of a circle of radius  $\rho$  centered at  $(R, \phi)$  (see figure). Applying the cosine law to side  $\rho$  of  $\triangle OCP$ , we have

$$\begin{aligned} \rho^2 &= R^2 + r^2 - 2Rr \cos(\theta - \phi) \\ &= [r - R \cos(\theta - \phi)]^2 + R^2 - R^2 \cos^2(\theta - \phi) \\ \implies [r - R \cos(\theta - \phi)]^2 &= \rho^2 - R^2 \sin^2(\theta - \phi). \end{aligned}$$



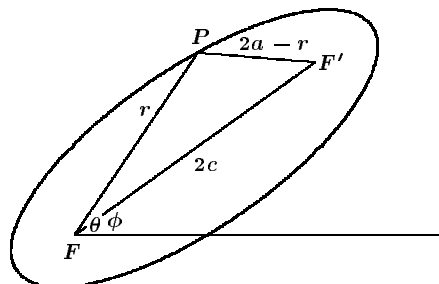


From this equation, it is evident that if  $R \geq \rho$ , then the curve is defined for a limited range of  $\theta$  given by  $-\frac{\rho}{R} \leq \sin(\theta - \phi) \leq \frac{\rho}{R}$ , as we would expect when the origin lies outside the circle. The squared length of the tangent from  $O$ , when  $\sin(\theta - \phi) = \frac{\rho}{R}$ , is  $P = R^2 \cos^2(\theta - \phi) = R^2(1 - \frac{\rho^2}{R^2}) = R^2 - \rho^2$ ; this is called the

*potence* of the origin w.r.t. the circle. Note that this formula is correct, even when  $R < \rho$  and  $\sin \theta = \frac{\rho}{R}$  has no solution. Incidentally, the solution to Exercise 2 can be obtained easily by noting that if the origin is on the circle, then  $R = \rho$ , and

$$r - R \cos(\theta - \phi) = R \cos(\theta - \phi) \implies r = 2R \cos(\theta - \phi).$$

**Example 3.** Polar equation of the ellipse with one focus at the origin and the main axis at an angle  $\phi$  to the polar axis. The cosine law applied to side  $F'P$  of triangle  $FF'P$  gives



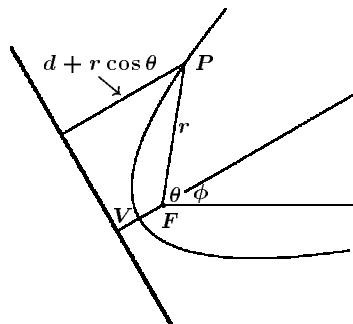
$$\begin{aligned} (2a - r)^2 &= r^2 + 4c^2 - 4rc \cos(\theta - \phi) \\ \implies a^2 - ar &= c^2 - rc \cos(\theta - \phi) \\ \implies r[a - c \cos(\theta - \phi)] &= a^2 - c^2 \\ \implies r &= \frac{b^2/a}{1 - e \cos(\theta - \phi)}. \end{aligned}$$

### A Catalogue of Important Curves

The following curves are all important in their own right, but since their polar expressions are particularly simple, they make good examples of the use of polar coordinates.

### Conic sections

We already have the equation for the ellipse. The polar equation of the parabola is even easier to derive. In the figure, we have the focus at the origin, the axis at an angle  $\phi$  to the polar axis and a distance  $d$  from the focus to the directrix. From the figure on the previous page, we have

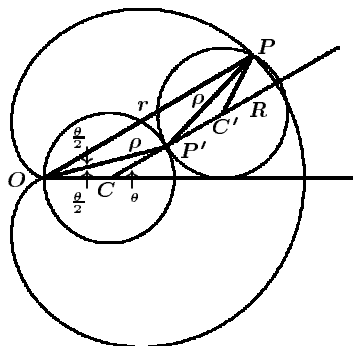


$$r = d + r \cos(\theta - \phi) \implies r = \frac{d}{1 - \cos(\theta - \phi)}.$$

**Exercise 3.** In a similar way, derive the equation for the hyperbola, noting how both branches are handled. Hence, deduce that the general equation of the conic is  $r = \frac{de}{1 - e \cos(\theta - \phi)}$ , where  $e$  is the eccentricity. This equation can be obtained immediately from the definition of a conic as the locus of the points whose distances to a line (called the *directrix*) and a point (called the *focus*) are at a constant ratio  $e$ .

### The Cardioid

The cardioid is the trajectory of a point on a circle that rolls on another circle of the same radius. So defined, it is a special case of the epicycloid, which is the curve described by a point on a circle rolling on another circle with no restriction on the radii; the general equation of the epicycloid is best expressed in parametric form.



In the figure, the two circles have radius  $R$ . The condition that the one centered at  $C'$  rolls on the one centered at  $C$  implies that triangles  $OCP'$  and  $PC'P'$  must be congruent. In triangles  $OCP'$  and  $PC'P'$ ,  $\rho = 2R \cos(\theta/2)$ . Similarly, in triangle  $OP'P$ ,  $r = 2\rho \cos(\theta/2)$ . Putting all together, we have

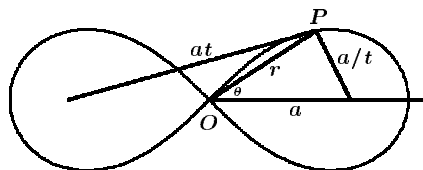
$$r = 4R \cos^2(\theta/2) \implies r = 2R(1 + \cos \theta).$$

The cardioid is also a special case of *Pascal's Limaçon*, of the equation  $r = b + a \cos \theta$ . The cusp at  $O$  becomes a loop if  $b < a$ , and a smooth indentation if  $b > a$ . The limaçon can be defined as the locus of the feet

of perpendiculars dropped from the origin to tangent lines to a circle. The radius of the circle is  $b$  and the distance from  $O$  to its centre is  $a$ .

### The Lemniscate

The lemniscate is the locus of the points such that the product of their distances to two points  $2a$  apart is  $a^2$ . In the figure, the cosine law gives



$$\begin{aligned} a^2 t^2 &= r^2 + a^2 + 2ar \cos \theta, & a^2/t^2 &= r^2 + a^2 - 2ar \cos \theta \\ \implies a^4 &= (r^2 + a^2)^2 - 4a^2 r^2 \cos^2 \theta \\ \implies r^4 &= 2a^2 r^2 (2 \cos^2 \theta - 1) \\ \implies r^2 &= 2a^2 \cos(2\theta). \end{aligned}$$

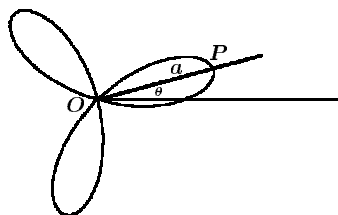
The lemniscate is a special case of *Cassini's ovals*, which are defined in the same way, but with no restriction on the product of the distances.

### The Rose

Loosely related to the lemniscate are the roses, of equation

$$r = a \cos[n(\theta - \phi)]$$

with integer  $n$ . For odd  $n$ , the curve has  $n$  'leaves' and it is traced completely when  $\theta$  varies from 0 to  $\pi$ . For even  $n$ , the curve has  $2n$



'leaves', and it is traced completely only when  $\theta$  varies from 0 to  $2\pi$  (see the figure).

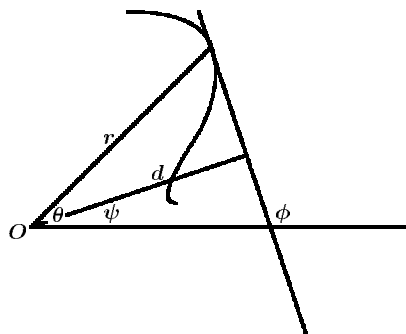
More general curves can be obtained if  $n$  is rational or irrational. In the first case there is an integer number of overlapping lobes and the curve is closed, but in the latter case the curve never closes, and in fact it is dense in the disc  $r \leq a$ .

### The Spirals

Polar coordinates are particularly suited to spirals. The two most famous are the Archimedean Spiral  $r = a\theta$ , which is the trajectory of a point whose angular and radial velocities are proportional, and the Logarithmic Spiral  $r = e^{a\theta}$ . In fact, any continuous monotonic function that goes to infinity as  $\theta$  goes to infinity defined in a semi-infinite interval will give rise to a spiral, like  $r = a \ln \theta$ . A related feature of polar curves is the *limit cycle*, occurring when  $r(\theta)$  has a finite limit  $r_0$  at infinity. In that case, the curve winds around the origin infinitely many times, approaching the circle  $r = r_0$ . Recognizing a limit cycle makes it easier to sketch a polar curve.

### Straight Lines

We finish by deriving the equation of a straight line not passing through the origin. From the figure, we have  $d = r \cos(\theta - \psi)$ , where  $d$  is the distance from the line to the origin and  $\psi$  is the direction of the closest point. An alternative form is  $d = r \sin(\theta - \phi)$ , where  $\phi$  is the direction of the line and  $0 > \phi - \theta > \pi$  (see the figure).



### Additional Problems

**Problem 1.** Considering  $r(\theta_0 + \alpha)$  and  $r(\theta_0 - \alpha)$ , derive an expression for a secant line to a conic. Passing to the limit  $\alpha \rightarrow 0$ , write an equation for the tangent line at  $\theta_0$ .

**Problem 2.**  $PQ$  is a chord through the focus  $F$  of a conic, and the tangents at  $P, Q$  meet at  $T$ . Prove that  $T$  lies on the directrix corresponding to  $F$  and that  $FT \perp PQ$ .

**Problem 3.** Let  $s$  be the tangent line at the vertex of a parabola and  $t$  be the tangent at  $P$ . If  $r$  and  $s$  meet at  $Q$ , prove that  $FQ$  bisects the angle between  $FP$  and the axis of the parabola, and that  $FQ \perp s$ .

## IMO Report

Richard Hoshino

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After ten days of intensive training at the Fields Institute in Toronto, the 1996 Canadian IMO team travelled to Mumbai, India to participate in the 37th International Mathematical Olympiad. For the first time in our team's history, every team member brought home a medal, with three silver and three bronze.

This year's team members were: Sabin "Get me a donut" Cautis, Adrian "Da Chef" Chan, Byung Kyu "Spring Roll" Chun, Richard "YES! WE'VE GOT BAGELS!" Hoshino, Derek "Leggo my Eggo" Kisman, and Soroosh "Mr. Bean" Yazdani. Our team leaders were J.P. "Radishes" Grossman and Ravi "Oli" Vakil (no, he's not Italian). Special thanks go out to our coaches, Naoki "Dr. Cow" Sato and Georg "Where's my Ethanol" Gunther.

This year's paper was one of the most difficult ever, and thus, the cutoffs for medals were among the lowest in history, 28 for gold, 20 for silver and 12 for bronze. Only one student, a Romanian, received a perfect score of 42. Our team's scores were as follows:

CAN 1	Sabin Cautis	13	Bronze Medal
CAN 2	Adrian Chan	14	Bronze Medal
CAN 3	Byung Kyu Chun	18	Bronze Medal
CAN 4	Richard Hoshino	22	Silver Medal
CAN 5	Derek Kisman	22	Silver Medal
CAN 6	Soroosh Yazdani	22	Silver Medal

Some weird coincidences: all the silver medallists got the exact same score, are graduating and are headed to the University of Waterloo in September, and all the bronze medallists are eligible to return to Argentina for next year's IMO. Overall, Canada finished sixteenth out of seventy-five countries, one of our highest rankings ever. A main reason for our success was our combined team score of 36 out of 42 on question #6, a problem that many countries answered very poorly (in fact, only two countries had more points on that problem than we did, and they both got 37 out of 42). Unfortunately, question #2 was answered very poorly by Canada, with only one 7, even though the problem was created by our own team leader, J.P. Grossman.

We all owe special thanks to Dr. Graham Wright of the Canadian Mathematical Society, Dr. Bruce Shawyer of Memorial University and Dr. Richard Nowakowski of Dalhousie University for their hard work and organization in making our trip possible and Dr. Ed Barbeau of the University of Toronto for all his commitment and dedication to training all the IMO team hopefuls with his year-long correspondence program.

Overall, the experience was memorable for all of us, although we could have done without the cockroaches in our rooms. Best of luck to all the students who will be working hard to make the 1997 IMO team, which will be held in Chapadmalal, Argentina near Mar del Plata.

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## Mayhem Problems

A new year brings new changes and new problem editors. *Cyrus Hsia* now takes over the helm as *Mayhem Advanced Problems Editor*, with *Richard Hoshino* filling his spot as the *Mayhem High School Problems Editor*, and veteran *Ravi Vakil* maintains his post as *Mayhem Challenge Board Problems Editor*. Note that all correspondence should be sent to the appropriate editor — see the relevant section.

In this issue, you will find only problems — the next issue will feature only solutions. We intend to have problems and solutions in alternate issues.