Mathematical / Computational Finance
Finance mathématique et computationelle
(Org: **Ruppa K. Thulasiram** (Manitoba))

RAJ APPADOO, University of Manitoba
Lattice Valuation Model Using $O(m,n)$-Trapezoidal Type Fuzzy Numbers

In this paper we discuss an option valuation model using $O(m,n)$ trapezoidal type fuzzy numbers. We demonstrate how fuzzy algebra assisted by $O(m,n)$ trapezoidal type fuzzy numbers can be successfully applied to the discrete Cox–Ross–Rubinstein (1979) binomial risk neutral option pricing model. Through option pricing theory and fuzzy set theory we get results that allow us to effectively price option in a fuzzy environment. The approach developed in the paper is illustrated with the help of a numerical example.

Joint work with R. Thulasiram, C. R. Bector and S. K. Bhatt.

JOE CAMPOLIETI, Wilfrid Laurier
New State Dependent Diffusions and their Application to Option Pricing

In recent years a new methodology (called "diffusion canonical transformation") was developed for generating new families of analytically solvable one-dimensional Markovian diffusions with multiple adjustable model parameters. In this talk we discuss some of the most recent, and ongoing, developments and applications of such new diffusion models to option pricing. We show that certain subfamilies of these processes properly describe forward (discounted) asset prices as martingales under a risk-neutral measure. Risk-neutral transition densities, first-passage time densities, and option prices for standard Europeans, barrier options and lookback options are derived in analytically closed-form. The newly derived analytical formulas for these families are supersets of previously derived formulas given for other popular models such as the CEV and others. Implied volatility surfaces for these models, however, exhibit a wide range of pronounced smiles and skews of the type observed in the option markets. These diffusion models are also applicable to pricing more generally path-dependent as well as multi-asset options. We conclude by discussing some numerical implementations for pricing Asian and Bermudan options via path integral approaches.

TAHIR CHOULLI, University of Alberta
Two optimal pricing measures: A comparative study

In this talk, we will discuss two optimal martingale measures: The minimal Hellinger martingale measure of order q ($\mathrm{MHM}(q)$ measure hereafter) and the q-optimal martingale measure, for any q and any semimartingale market model.

We will explain how these two optimal martingale measures are obtained. Then we conduct our comparison for these two martingale measures in two ways. The first comparison deals with comparing 'physically' the two optimal martingale measures. Precisely, in this case, we show that the two optimal martingale measures coincide in the case of Levy market models with known horizon, while they differ in general. We also provide necessary and sufficient conditions for the two optimal martingale measures to coincide in a general framework. The second comparison addresses the question whether there exists a model for which the $\mathrm{MHM}(q)$ measure of the underlying model is the q-optimal martingale measure, and vice-versa. This last comparison has a great impact in analysing uncertainty models. Finally, from the very practical point of view, we analyze the $\mathrm{MHM}(q)$ measure for the case of discrete-time market model.

HAOHAN HUANG, York University, 4700 Keele Street, Toronto, ON
Estimating Value at Risk with Non-negative Matrix Factorization technique

In this talk I will introduce the Non-negative Matrix Factorization (NMF) technique to estimate Value at Risk (VaR). VaR is a very important methodology for measuring portfolio risk in finance. Normally when calculating VaR, we need the correlations between each product in our portfolio. But it's very time-consuming to get the correlations when the number of products is large, besides, the correlation cannot be very precise. The Non-negative Matrix Factorization method has previously been shown to be a useful decomposition for multivariate data. It is developed now to find parts-based, linear representations of non-negative data. Then I use this tool to deal with the data of the portfolio and gladly find I can skip the step of calculating correlations when estimating VaR.

ALI LAZRAK, University of British Columbia
Perfect competition among generations: growth theory under time

This paper characterizes differentiable subgame perfect equilibria in a continuous time intertemporal decision optimization problem with non-constant discounting. It is assumed that successive decision-makers are in a situation of perfect competition, so that each of them takes the strategy of future generations as given. We show that equilibrium strategies are characterized by a value function, which must satisfy a certain equation. The equilibrium equation takes two different forms, one of which is reminiscent of the classical Hamilton–Jacobi–Bellman equation of optimal control, but with a non-local term. We give a local existence result, and several examples of equilibria, and we conclude that non constant discount rates generate an indeterminacy of the steady state in the Ramsey growth model. Despite its indeterminacy, the steady state level is robust to small deviations from constant discount rates.

Co-author: Ivar Ekeland.

ALEX PASEKA, University of Manitoba
Information in Option Prices and the Underlying Asset Dynamics

We jointly estimate stochastic volatility (Heston) model parameters in both the physical and equivalent martingale measures, by exploiting all the information in a broad cross-section of option prices along with the dynamics of the underlying asset, using the exact probabilistic framework of the model. To this end, we derive the necessary joint transition density to draw from the latent volatility process conditioned on the observed returns. We answer questions along two dimensions. First, no study to date has used all the information in the exact transition densities from the stochastic volatility (Heston) model. Second, in the context of efficient estimation, how much of the mispricing may be attributable to parameter and state variable uncertainty? Finally, our metric for assessing the fit of the model is the predictive posterior on the implied volatility smile. Using this metric we assess directly the model's ability to capture the empirical smile, which is an important criterion in evaluating an option pricing model.

For the deep in-the-money, short-term call, parameter and state variable uncertainty give rise to a 90%ile band in the predictive posterior density of the implied volatility of 0.56%. But the data are on average 3.6% away from the predictive posterior density mean. By contrast a slightly in-the-money, intermediate term option has average pricing errors relative to the posterior mean of 0.26%, and parameter and state variable uncertainty imply that the predictive posterior density's 90%ile band width is 0.58% (suggesting an excellent model fit).

LUIS SECO, University of Toronto, Department of Mathematics, 40 St. George Street, Toronto M5S 3G3
Modeling stochastic correlation in financial markets

This talk will review some old and new models of stochastic correlation, with applications to the pricing and risk sensitivities of some correlation-sensitivie financial instruments, such as spread options and CDO's.

LIQUN WANG, University of Manitoba, Department of Statistics, Winnipeg, Manitoba, R3T 2N2
First Hitting Time Distribution for Diffusion Processes and Time-Dependent Double Barriers

The first hitting time distributions (or boundary crossing probabilities) play an important role in pricing barrier options and other financial derivatives. In particular, evaluation of time-dependent barrier options leads to the first hitting time distribution for nonlinear boundaries. Because no explicit formula exists in such situations, the entailed numerical evaluation is a difficult task. The computation of boundary crossing probabilities arises also in many other scientific fields, e.g., in biology, epidemiology, econometrics and statistics.

In this talk, I will present explicit formulae for the probabilities that a Brownian motion crosses piecewise linear boundaries. Then I will use this formula to approximate the crossing probabilities for general nonlinear boundaries. This technique is further extended to a class of more general diffusion processes, including Ornstein–Uhlenbeck processes and geometric Brownian motion with time-dependent drift. The numerical computation is done using Monte Carlo integration which is straightforward and easy to implement. Some numerical examples will be presented to illustrate this technique.

XIAONAN WU, Memorial University of Newfoundland,St John's, NL
Anomaly Detection in Financial Fraud Data

Building effective fraud detectors can significantly reduce billions of losses of financial institutions due to fraudulent activities. One popular anomaly detection method is to check changes in users' behavior that might hint at suspicious activity. Boundary that distinguishes normal and abnormal space determines detection accuracy. In this talk, we will discuss how to accurately define this boundary inspired by an interesting natural phenomenon: species in nature undergo intensive competitions and interactions with environment, and finally come into balance. In our approach, we define two sets of rules: positive rules for normal space and negative rules for abnormal space, and refer each of them as a species, which evolves independently. Meanwhile they control each other's evolutionary environment, such as selection pressure and crossover/mutation rates. During the evolutionary process, positive rules and negative rules will move towards the boundary. Any rule which is over the boundary will be punished. In the end, both of the rule sets will converge around the boundary. Hence the boundary between normal and abnormal space can be accurately defined, thus helps to avoid the over-generalization problem which exists in methods that consider normal space only and "hole" (insufficient detectors) problem which is caused by considering abnormal space only. We have applied this method to real financial data to detect fraud. The preliminary results indicate that this approach provides good accuracy and is able to scan financial databases quickly.