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First Hitting Time Distribution for Diffusion Processes and Time-Dependent Double Barriers

The first hitting time distributions (or boundary crossing probabilities) play an important role in pricing barrier options and other financial derivatives. In particular, evaluation of time-dependent barrier options leads to the first hitting time distribution for nonlinear boundaries. Because no explicit formula exists in such situations, the entailed numerical evaluation is a difficult task. The computation of boundary crossing probabilities arises also in many other scientific fields, e.g., in biology, epidemiology, econometrics and statistics.

In this talk, I will present explicit formulae for the probabilities that a Brownian motion crosses piecewise linear boundaries. Then I will use this formula to approximate the crossing probabilities for general nonlinear boundaries. This technique is further extended to a class of more general diffusion processes, including Ornstein–Uhlenbeck processes and geometric Brownian motion with time-dependent drift. The numerical computation is done using Monte Carlo integration which is straightforward and easy to implement. Some numerical examples will be presented to illustrate this technique.