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Integer Parts and Complements to Valuation Rings of Ordered Fields
An integer part (IP for short) $Z$ of an ordered field $K$ is a discretely ordered subring, with 1 as the least positive element, and such that for every $x \in K$, there is a $z \in Z$ such that $z \leq x<z+1$. Mourgues and Ressayre establish the existence of an IP for any real closed field $K$ by showing that there is an order preserving embedding $\varphi$ of $K$ into the field of generalized power series $k((G))$ such that $\varphi(K)$ is a truncation closed subfield (here $k$ is the residue field and $G$ the value group of $K$ ). An IP of $K$ obtained in this way (i.e., from a truncation closed embedding) is called a truncation integer part of $K$. IPs appear naturally in model theoretic arithmetic, algebra and analysis; e.g. Shepherdson showed that IPs of real closed fields are precisely the models of a fragment of Peano Arithmetic called Open Induction, whereas truncation IPs played a crucial role in Ressayre's investigations of the model theory of the real exponential field. In this talk, we analyze IPs from a valuation theoretic viewpoint and summarize their main special features. We investigate their connection to special (additive) complements of valuation rings of ordered fields. This approach reveals new interesting valuation theoretic properties of arbitrary valued fields (not just ordered fields); depending on whether such special complements exist. We discuss these properties and their implications, thereby giving an intrinsic valuation theoretic interpretation of truncation closed embeddings in fields of power series.
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