String Theory and Integrable Systems
Théorie des cordes et systèmes intégrables
(Org: **Lisa Jeffrey** (Toronto), **Boris Khesin** (Toronto) and/et **Rob Myers** (Perimeter Inst.))

YULY BILLIG, Carleton University
Representations of extended affine algebras and hierarchies of non-linear PDEs

Extended affine Lie algebras are higher rank generalizations of affine Kac–Moody algebras. In this talk I will discuss recent developments in the representation theory of these algebras, and their connections with hierarchies of soliton PDEs.

FREDDY CACHAZO, Institute for Advanced Study
One-Loop Amplitudes of Gluons in $N = 4$ super Yang–Mills

Very recently several new techniques in perturbative gauge theory have been introduced. At one-loop, any amplitude of gluons in $N = 4$ super Yang–Mills can be written as a linear combination of known scalar box integrals with coefficients that are rational functions. Using a generalization of unitarity cuts, in particular quadruple cuts, any coefficient can be easily written as the product of four tree-level amplitudes. Therefore, this new technique solves the problem of computing one-loop amplitudes in $N = 4$ super-Yang–Mills.

EMANUEL DIACONESCU, Rutgers University
A Vertex Formalism for Local Ruled Surfaces

[no abstract]

JACQUES DISTLER, Physics Department, University of Texas at Austin, Austin, TX 78712
Chiral Rings for $(0, 2)$ Models

The Topological A-Model is a "twisted" version of the $N = (2, 2)$ supersymmetric σ-model, with target space, X. The ring of observables of the A-Model is isomorphic to a certain deformation of the cohomology ring of X. I would like to present a generalization of this structure to the case of $N = (0, 2)$ supersymmetry. The data will consist of X and a rank-r holomorphic vector bundle $V \to X$, satisfying $\bigwedge^r V = K_X$.

I will explain, first, from the point of view of the twisted supersymmetric σ-model, why a finite-dimensional graded-commutative ring exists. And I will explain, in a few examples, how quantum effect deform the ring structure.

This is joint work with Allan Adams and Morten Ernebjerg.

DMITRI KOROTKIN, Concordia University, 7141 Sherbrooke West, Montreal, Quebec H4B 1R6
Tau-functions on spaces of abelian differentials and determinants of Laplacians in flat metrics with conical singularities over Riemann surfaces

We define a natural analog of the Jimbo–Miwa tau-function on different strata of the space of holomorphic differentials over Riemann surfaces. We compute the tau-functions in terms of higher genus generalization of Dedekind eta-function. The developed formalism is applied to rigorously compute the determinants of Laplace operators over Riemann surfaces in flat metrics with conical singularities. The holomorphic factorization formula for such determinants gives the higher genus generalization of genus one expression by Ray–Singer.

This is a joint work with Alexey Kokotov.

SHAHN MAJID, Queen Mary, University of London
Semiclassicalisaton of quantum differentials and Poisson geometry

We semiclassicalise the standard notion of differential calculus in noncommutative geometry on algebras and quantum groups. We show in the symplectic case that the infinitesimal data for a differential calculus is a symplectic connection, and interpret its curvature as lowest order nonassociativity of the exterior algebra. In the Poisson–Lie group case we study left-covariant infinitesimal data in terms of partial connections. We show that the moduli space of bicovariant infinitesimal data for quasi-triangular Poisson–Lie groups has a canonical reference point which is flat in the triangular case. Using a theorem of Kostant, we completely determine the moduli space when the Lie algebra is simple: the canonical partial connection is the unique point for other than $\mathfrak{sl}_n$, $n > 2$, when the moduli space is 1-dimensional. This proves that the deformation-theoretic exterior algebra on standard quantum groups must be nonassociative and we provide it as a super-quasiHopf algebra. More generally, we show that many standard quantisations in physics including of coadjoint orbits (such as fuzzy spheres) have naturally nonassociative differential structures. Our methods also quantise quasi-Poisson manifolds of interest in string theory.

Mostly joint work with E. J. Beggs.

GERARD MISIOLEK, Mathematics, University of Notre Dame, IN 46556, USA
Recent well-posedness results for the CH equation

I will describe some recent results on analyticity and ill-posedness.

ANDREW NEITZKE, Harvard University, Cambridge, MA
BPS Microstates and the Open Topological String Wave Function

I will describe recent joint work with Mina Aganagic and Cumrun Vafa, which reinterprets the square of the open topological string wave function (also known as the generating function for open Gromov–Witten invariants) in terms of counting supersymmetric microstates localized on a stringy defect in a gravitational theory in 4 dimensions. I will also sketch the sense in which the wave function property of the topological string, which plays a crucial role in this work, is related to integrability.

RONEN PLESSER, Duke University
Linear Sigma Models and Coulomb Branches

[no abstract]

MATSUO SATO, Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627-0171, USA)
Integrability of the $\mathrm{Ad}\, S_5 \times S^5$ *Superstring*

We study integrability aspects of superstrings on $\mathrm{Ad}\, S_5 \times S^5$. We show that a one parameter family of flat currents, which is gauge equivalent to that obtained by Bena, Polchinski and Roiban, is manifestly invariant under a generalized Z_4 transformation. This symmetry is expected to simplify analysis of the currents because the Z_4 transformation is an automorphism of $\mathrm{PSU}(2,2|4)$, the isometry in the theory.

We perform the canonical analysis of the theory. Especially we calculate the Poisson bracket of the currents. This bracket results in an algebra which includes a Schwinger term. Because of the Schwinger term, more work is needed in understanding the quantum integrability properties of the system.

MICHAEL SHAPIRO, Michigan State University, East Lansing, MI 48823
Cluster algebras and Poisson Geometry

We describe a Poisson structure compatible with a cluster algebra structure. In particular case of cluster algebra formed by Penner coordinates on the decorated Teichmuller space that leads to a known Weil–Petersson symplectic form a Teichmuller space.

JACEK SZMIGIELSKI, Department of Mathematics and Statistics, University of Saskatchewan, 106 Wiggins Rd., Saskatoon, S7N 5E6
Degasperis–Procesi peakons and the discrete cubic string

We use an inverse scattering approach to study multi-peakon solutions of the Degasperis–Procesi (DP) equation, an integrable PDE similar to the Camassa–Holm shallow water equation. The spectral problem associated to the DP equation is equivalent under a change of variables to what we call the cubic string problem, which is a third order non-selfadjoint generalization of the well-known equation describing the vibrational modes of an inhomogeneous string attached at its ends.

For the discrete cubic string (analogous to a string consisting of n point masses) we solve explicitly the inverse spectral problem of reconstructing the mass distribution from suitable spectral data, and this leads to explicit formulas for the general n-peakon solution of the DP equation. Central to our study of the inverse problem is a peculiar type of simultaneous rational approximation of the two Weyl functions of the cubic string, similar to classical Padé–Hermite approximation but with lower order of approximation and an additional symmetry condition instead. The results obtained are intriguing and nontrivial generalizations of classical facts from the theory of Stieltjes continued fractions and orthogonal polynomials.

This talk is based on joint work with Hans Lundmark (Linköping University, Sweden) which, under the same title, appeared recently (International Mathematics Research Papers, vol. 2005, 2, 53–116).

KIRILL VANINSKY, Michigan State University

Poisson structures on meromorphic functions defined on Riemann surfaces and classical integrable models

In 1988 Atiyah and Hitchin introduced a Poisson bracket (PB) on meromorphic functions defined on the Riemann sphere.

Can one replace the Riemann sphere by a Riemann surface of genus $g > 0$? Are there other natural Poisson structures?

We survey recent progress in these problems. It is based on the theory of classical completely integrable systems.