Problem Solving as Motivation in Mathematics:

Just in Time Teaching

Jeff Irvine, Brock University

Students work on a problem involving profit maximization for the school store. What mathematics concepts are they learning? Why don't they ask "When are we ever going to use this, sir?"

This vignette describes a program based on the philosophy that the utility of mathematics must be clear and explicit to every student. The program was based on two tenets: every class began with a problem, thus motivating students to engage; and just in time teaching, whereby content was taught as needed to resolve the problem. Throughout this program, there was not a single report of a student asking "When are we ever going to use this, sir?". By placing the mathematics content in a problem solving context, the question of utility was never an issue.

Structure of the Program

Class Problem. Every class started with a problem. As far as possible, the problem would be based on real world concepts. The working definition of real world involved one or more of these conditions: students could use the mathematics immediately, for example, in their part time jobs, budgeting, or sports; students could use the mathematics in another subject, in the near term, such as in science, geography, technical shops, family studies; someone close to the student could or did use the math content, such as a family member, relative, adult acquaintance; there were examples in the real world of people using the mathematics; the mathematics flowed from an investigation, experiment, or model in which the students were involved. What was explicitly not accepted was the traditional "trust me, you'll use this later". The problem might be the focus of one class, or an over-reaching problem might cover multiple classes, with each class dealing with a sub problem of the main problem. The class problem was explicitly designed to go beyond the students' current knowledge, or to use that knowledge in a new way. Student motivation to study the content was paramount to this program. Every class was designed to give the students an immediate reason to learn the mathematics content. Some examples of class problem topics are shown in Table 1. The problems ranged from closed problems, with a predictable problem solving route and a single answer, through open routed problems with a single answer but multiple paths to the solution, and included some truly open problems, with multiple solution pathways and multiple possible answers.

Topic of Study	Problem Focii
Venn diagrams	Earthquake epicentres; consumer attributes
Intersection of lines	Breakeven analysis; pursuit problems
Integers	Temperature, especially extremes; climate (geography)
Logarithms	Richter scales; pH; magnitudes of stars
Linear relations	Simple interest; comparing printing costs; density (science)
Bar graphs	Climate; population (geography)
Circle graphs	Net worth; budgeting

Exponential growth	Population; compound interest
Quadratic functions	Profit maximization; optimization problems; headlights; catenaries
Hyperbolas	LORAN navigation system; comets
Ellipses	Planetary orbits; satellite transfer ellipses
Perimeter, area, volume	Fence it, paint it, fill it up; design a garden, room, amusement park
Similar triangles	Inaccessible distances; shadows
Triangle trigonometry	Inaccessible heights; clinometers
Displacement, velocity, acceleration	Physics problems; experiments
Periodic functions	Radio waves; biorhythms; Ferris wheels
Geometric sequences and series	Compound interest; annuities; chessboard problems
Arithmetic sequences and series	Simple interest; linear relations
Matrix operations	Power ratings of sports teams; Markov chains; communication networks; cryptography
Matrix equations	Leontiev production models; Kirchoff's laws; election predictions; consumer behaviour
Equations	Formulas such as D=ST; D=M/V; V=IR; SP=(1+P%)CP
Systems of equations	Mixtures; puzzles; DST; money; percents

Table 1. A sample of content topics and related class problem topics.

Just In Time Teaching. The second dimension of this program was Just In Time Teaching. The just in time concept comes from industry, where it was pioneered in North America by companies like Toyota and Dell Computers The just in time structure is based on providing customers with supplies "just in time", that is, exactly when the supplies are needed. So, a Toyota car plant would keep very low levels of parts inventories, and their sub suppliers would provide parts very near to the time the Toyota plant needed them. In this way, Toyota reduced their inventory carrying costs, as well as reducing their average ordering costs. In teaching, the JIT concept was modified to support the problem solving focus. For example, if a problem solution required knowledge of equation solving, and equation solving had not already been encountered by the students, a lesson or mini-lesson on equation solving would be taught. After practice and consolidation, the students would return to the class problem. If equation solving was the only skill needed to complete the problem, the students would then use the new skill to complete their solutions. If additional new skills or concepts were needed, another pause would occur, while a lesson was taught on these additional skills. This process would continue until the class problem was completed. By using this structure, it was always clear to the students why they were learning the mathematics content "just in time".

Example of Just In Time Teaching. The class problem involved breakeven analysis for a student-run enterprise, such as the school yearbook. Information was provided on production fixed and variable costs, as well as probable consumer demand and selling prices. The expected mathematics content was intersection of lines. JIT teaching lessons or min-lessons would include: algebraic formulation of equations; linear relations and graphing; solving systems of equations by multiple methods; special cases, such as parallel or coincident lines; role play, such as corporate CEO; communications, such as letters to suppliers recommending a course of action;

extensions into quadratic relations to consider concepts such as profit, or the inverse relation between price and consumer demand. All the lessons would be supported by practice and consolidation. The student discussion might be extended to involve What If? questions, or research to find other real life uses of the math content.

Groupings. Each classroom contained student desks organized in pairs. A common teaching technique involved a whole class discussion of the class problem, setting the stage for student activity, followed by students working on the problem in pairs or groups of four. Some teachers, who were more comfortable with whole class activities, allocated the major portion of class time to solving the class problem as a whole class. Other teachers utilized a mixture of whole class and pairs or groups. One teacher frequently allowed students to attack the class problem as pairs, without any initial whole class discussion. Depending on the topic, additional student activities such as role play, library research, or communication activities such as letter writing to argue a position could be used to enhance student learning.

Technology. Available technology was routinely used as problem solving tools. The typical technology of the day was scientific calculators. Students were allowed to use calculators at any time in their work. This allowed the problems to be much more real life, since, in real life, problems are often "messy" and involve messy, unfriendly numbers. Using calculators also freed students from using tables of, for example, trigonometric ratio values, or logarithm tables. An important part of using technology appropriately was to teach students good estimation skills, and imbue them with an "estimate, then calculate" mentality. This philosophy on the use of technology at any time was controversial at the time. The department head faced vociferous complaints from parents, and attended a number of school council meetings to defend the department's position. As a teacher in the department, my reaction to the technology use was that it was incredibly freeing. No longer did I have to invent problems with "nice" numbers, or interest rates that were tabulated in the back of a textbook. The problems could actually use "real world" data.

Origin of the Program

The department head of the mathematics department was a former aeronautical engineer, who went into teaching when the jet aircraft on which he was working (Avro Arrow) was cancelled by the Canadian government. He had a very strong philosophy of mathematics as a tool, and that students needed to see the relevance of their learning through relating the content to the real world. He was a visionary, foreshadowing the problem solving focus now so prevalent, by over 20 years.

When opening a new secondary school. the department head hand-picked a department who were in agreement with his philosophy, and agreed as a condition of moving to the school, to reverse the traditional theory followed by application, into application followed by theory necessary to solve or move forward the real world problem. I was a member of this department.

Summary

Several of the teachers recalled students frequently asking before class "What's the CP (class problem) for today, sir?" Within a few weeks or months, student ownership of the program was very high. Student engagement and time on task during class was extremely high. However, the program was not without difficulties. Creating realistic class problems, especially for many topics in algebra, resulted in "pseudo-problems", such as area models for multiplication of binomials. As a consequence, the real-life orientation of the problems was sometimes violated. Also, this program was difficult for teachers to maintain. Teaching through problem solving involves a high degree of commitment by the teacher, high teacher self efficacy, and necessitates frequent, thoughtful interactions by the teacher with the students, supporting them in the problems solving process. As the school grew in size, and more teachers were added to the department, it became more difficult to maintain a cohesive problem solving focus. While the original department members strove to remain faithful to the program, newer teachers, who did not necessarily own the philosophy espoused by the original members, sometimes chose alternative approaches to instruction. As teachers moved on to other locations, and the department head retired, the dedication to the original program was significantly reduced. In addition, as educational research in problem solving increased, other, possibly more effective research-affirmed instructional strategies emerged. In problem solving, increased emphasis on problems that facilitated far transfer have become more prevalent. However, this program, which was instituted over 30 years ago, served as a lighthouse program, illustrating an instructional possibility that addressed student learning much more broadly, and across dimensions beyond a single focus on knowledge acquisition, than was extant at that time. It was an honour to be a part of it.