Beautiful Dance Moves: Dancing the Transformations

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Drawing from the idea that physical experience changes how we process information, I have worked with a dance teacher to create a dance/math class that explores the language of transformations of functions using our bodies.

Beautiful Math Moves: Dancing the Transformations starts in Precalculus 11 when I take my classes outside to map out a representation of a quadratic function. In the classroom, any time a new function is introduced, we become the function and figure out how to illustrate different curves using our bodies. By Precalculus 12, most of my students are familiar with using their bodies to map out functions, and are comfortable moving into the dance studio.

We need to feel the mathematics before we describe it, so we start working with transformations in the dance studio. This embodiment of mathematics allows the students to make it personal before we move to abstraction. Math becomes less of something that is done in our heads, and more of something that is all around us.

The dance class starts with a warm up designed to introduce the students to the ideas of expansion and compression. Throughout the warm up we focus on counting the repetitions, changing the counts as we progress through the movements. This is also an opportunity to discuss frequency, which is later related to horizontal compressions and expansions of sinusoidal functions. Following the warm up, we play games with shapes and practice moving through space, all the time focussing on the idea of expanding/compressing into and away from our body's core.

The class is a dance class, with math being discussed. We finish the class by having students work in groups to create an actual dance featuring expansions and compressions. As the groups are practicing, the dance teacher and I move throughout the room, planning on how to incorporate the individual dances into an entire piece. We run through the dance several times, performing for an imagined audience.

After the dance class, we spend time mapping out functions on a grid as a group. We link arms and form different curves, expanding and compressing away from either axis, or both of them at the same time. Later, when we are considering operations on algebraic functions, we map out functions and try to step into the correct positions to show what would happen if we add a linear function and a square root function, or what might happen if we take the square root of a quadratic function.

Throughout the activities, many questions come up about the mathematics that don't typically emerge in a math class. Why are we quiet when compressing and noisy when expanding? This leads to a discussion of a vertical stretch being related to amplitude of a sine wave and the intensity of sound. A vertical expansion leads to an increase in amplitude, which results in a louder sound, while a vertical compression leads to a decrease in amplitude and a quieter sound. We discuss time in music relating to repetition. This is something that makes sense to the students immediately, which then leads to a discussion of a horizontal stretch relating to the frequency of a function. As the music gets faster, there is more repetition and the frequency increases which is a horizontal compression of a function. With a horizontal expansion, the frequency decreases and the music slows down. The students start to see a connection between the math we discuss in class and what is happening with the sounds they hear, strengthening their understanding of the abstract mathematical concepts.

There is a possibility to explore our senses and emotions: What emotions are associated with expansions versus compressions? This brings math into the realm of the subjective for students, many of whom associate mathematics with only the objective. We play, create and improvise, things that I want the students to bring with them from the studio into the classroom.

Beautiful Math Moves
(a 2-hour dance class)

1. **On the floor warm up: fetal position and banana to introduce the idea of expansion and compression.**

The floor is a good place to start as the students are more isolated and insular on the floor. It feels as if no one can see them, so there is less of a chance to become self-conscious and helps get rid of any inhibitions. Down is a gentler place to start, and there are fewer directions to move than when standing up.

Positions to move through:

sea star–laying on the back, with arms and legs outstretched into a sea star shape, relaxing into the floor. This is the neutral position.

fetal–from the sea star position, turn to one side and draw the arms and legs toward one another into a fetal position.

banana–from the fetal position, stretch the arms and legs away from one another while still on the side. Arms are together and legs are together, slowly arching the back and stretching the limbs outwards into the shape of a banana.

Run through these positions with a eight-count for each shape:

sea star
fetal, banana, fetal on the right
sea star
fetal, banana, fetal on the left
sea star

Do a couple of 8-counts as students get used to the shape changes, then switch to 4-count and then to 2-count.

* changing the count relates to a frequency change, which is related to a horizontal stretch later
* horizontal stretch = frequency (fast is a compression and slow is an expansion)

After the fetal/banana shape changing, move into leg swings. Still lying on the back, swing one leg at a time from left to right in the following way:

- swing to the left with the right leg
- keep the back on the floor, with the arms stretched out to the sides
- bend the left leg so that the right leg can pass over the left without getting caught
- repeat to the right with the left leg

Start with a 4-count swing and repeat a few times, and then switch to a 2-count. Use the belly to swing the legs, which will help to swing the legs faster and smoother.

2. Small shape/big shape game

This introduces movement from the core, a spot just below the navel. The brain is where we think from, the heart is where we feel from and the core is where we move from. Dancers move from their core, and functions are changed from the origin of the graph, linking the origin of a coordinate grid to the our core.

It is important to make note of breathing: use the breath to bring expansions and compressions into the entire space. As students move from 2-dimensional movements (on the floor) to 3-dimensional movements in space they may need to be encouraged to use the space and to move away from flatness.

Small shape/big shape:

- Start on the ground and gradually move away from the floor.
- Make a big shape with both feet and both hands on the floor and hold.
- Make a small shape with both feet and both hands on the floor and hold.
- Continue making big shapes and small shapes, but progress from all four hands and feet on the floor to three, one of each, through to only one foot.
- Move back and forth between small shapes/big shapes.

Encouragement is needed to help students experiment with different shapes.

* small shapes tend to be quiet while large shapes tend to be noisy, which relates to amplitude and the intensity of sound
* vertical stretch = amplitude/intensity (loud is an expansion and quiet is a compression)

3. Move around the room with music.

The words we use for transformations such as compress, expand and flip are doing words. Here is a chance to start doing these transformations with our bodies.

Students dancing for the first time will tend to move with their arms rather than their entire bodies. Remind them to move from the core, not just with their arms. Everything is expanding and compressing from this one place. This relates to stretching from/to the axes/origin of the coordinate grid, rather than any particular place on the function. The whole curves stretches, just as the whole body is stretching.

Walking through space:

Have the students walk around the room, avoiding one another and avoiding walking in a large circle. This is like skiing in trees: look for space to avoid the tress. Or, it's like shooting on goal in soccer: aim for the space, not the goalie.

- Walk naturally, heel-toe, arms swinging, neutral.
- Stop, and make small/big shapes in one space without walking: move through big, medium, small, medium, big
- Start walking through space again, making shapes while walking, and moving through the expansions and compressions

Have music playing and have students move through the shapes with a four-count. This keeps the movement of the individuals in time with one another.

4. Diagonals across the room

This starts to turn the movement into a dance, and is a typical dance class routine. Have the students in one corner of the room in pairs. Each pair moves across the room to the diagonal corner, and then back along the side to a third corner. Once all the students have moved through the diagonal, start over again, crossing the room across the other diagonal.

As the students are moving along the diagonal, have them expand and compress into big and small shapes, keeping time with the music.

This is a good time to introduce the idea of flips (most likely flipping from one side to the other, rather than from head to toe). Student can add in flips along the diagonal if feeling adventurous.

5. Group work: making dance pieces

In small groups, put together 4-5 movements, working as a group and focussing on expanding and compressing, with flips as a possibility.

Run through the piece three times with three different sizes: normal, as an expansion, and as a compression. It should be noticeable that the each time the movements are identical, but either smaller or bigger. Start without the music, but quickly add in the music so that they work on timing each movement to the music. All the groups should be working with the same timing, to make it possible to fit the different pieces together. For example, each movement should have four counts, so if a piece has four movements, the entire piece has 16 counts. Running through this three times will give 48 counts. Each group should have the same number of counts.

Extension:

While the groups are practicing their pieces, walk around and try to determine how each group could fit into one another's work. After the groups have each had a chance to perform their piece for the whole group, work on piecing them together so that by the end, there is an entire dance piece. It may be necessary to make subtle changes to the original pieces so that they blend together easily.

Another layer would be to an introduce an element of humanity. What emotions are associated with expansions, compressions, flips? Putting human emotions into the transformations introduces subjectivity into mathematics. How are you feeling today? What transformations reflect your mood? As the groups begin creating their pieces, ask them to consider these things and project their mood through the transformations they are choosing.

6. Move from the idea of stretching from/into our core, towards to/from the origin.

This section moves away from the dancing and into work with functions. It works well at the end of the dance class, but can also be used independently with a class. See the section **Other Activities Incorporating Movement in Math Class** for more ideas that can be used without the dance piece.

Functions have no centre (well, most of them don't!). The core is on the axis NOT the function. It's the function that moves, and the whole thing moves unless parts happen to be on an axis. Parts of the function will stay still depending on the position of the function in relation to an axis and the transformation that is being applied.

Tape vertical and horizontal axes on the floor.

Have students form a curve relating to a known function with some overlap on an axis. Give the students an instruction about a transformation: expand vertically by a factor of 2, compress horizontally by a factor of ½, flip around the x-axis, move to the right 3 space. Start with one transformation at a time and explain how each transformation works. For example, when expanding, move away from the axis and when compressing move towards the axis.

Work through various functions, expanding and compressing, flipping, moving right/left, up/down. Start to connect the movements with what is happening in a vertical direction and a horizontal direction.

Notes on the Dance Class

The idea of the Beautiful Math Moves came to me when I was working on my masters of arts education. One of the other students in the program was the dance teacher at the school I work at, and for one of her projects, she lead us through a dance workshop that followed the stages outlined above (without the function work). As she was explaining what we were going to do, she used words that were familiar from my math classes, and the idea of introducing dance into my math classes started forming in my head.

The stages in the dance class are typical to what might be experienced in a beginner's dance class. I have no dance training, and only feel comfortable leading my classes through this routine after working with the dance teacher with three different math classes. The routine should feel familiar to any dance teacher, and I would strongly recommend some time collaborating on this project. It was the collaborative process that the dance teacher and I worked through that gave voice to the ideas running through my head. She taught me about dance, and I taught her about mathematics. Her questions about the math forced me to be precise in my thinking, and her dance training allowed me to discover how to use dance in order to move towards a better understanding of the transformations.

Other Activities Incorporating Movement in Math Class

I recently had a conversation with a former student about all of the movement we did in math classes. She is currently studying engineering, and told me that she thinks of what we did with our bodies all the time. It was this movement that she credits with being able to move into 3-dimensional thinking in her engineering classes. Another student once told me that she could imagine doing the movements of the functions while she was writing a test, which helped her to deal with the questions on the paper in front of her.

It helps to have a large, open space available, although pushing desks and chairs back in the classroom will work as well. Mark an *x-y* axis on the floor with masking tape, or find a space with horizontal or vertical lines. I have used the lines on a tennis court and a soccer field, as well as creating a grid in the large, open multipurpose room in the centre of our school.

The Human Function Machine (Group Work)

Have students line up on a horizontal line representing the *x*-axis. Each student chooses an integer value and finds the right place to stand. You can place numbers on the floor, or just have the students say their number out loud. They will remember their own number and the numbers around them, and don't necessarily need a visual reminder of the number along the axis. These numbers are the inputs.

Give the students a rule: multiply your number by 2 and add 1. Find the output along the vertical axis.

The scale of the axis will depend on the space you are using. I ask students to practice taking one step forwards and backwards until everyone's steps are about the same size.

Once the outputs have been determined and the students are in their new positions, have the students discuss what shape they have formed. I use this to introduce linear functions and quadratic functions, and to help students feel the connection between exponential and logarithmic functions. Any time I introduce a function to a class, I try to find a way to map it out with the students.

One of the benefits is having students understand the shape of the functions that are formed with the different rules. While the students are stretched out in a parabola, I travel from the student on the "origin" and have the students tell me how many steps in a vertical direction I need to take from one student to the next. By the time we're back in the classroom, they are able to easily graph the function $y = x^2$ and can use the differences between the square numbers to help with the shape. Later, as we learn about stretching our parabolas, I will walk through the stretches in the classroom, and remind students of the movement along the function that we experienced outside.

It is also a good time to introduce what happens when we perform basic transformations: multiply the function by negative one; move the function to the right or up or down or to the left.

You can give a group of students some information about a function and ask them to map it out.

For example:

- become a line with a slope of 3 and a y-intercept of –2
- become a parabola that has been expanded vertically by a factor of 2
- become a cubic function that has been moved left two spaces

Once the students have mapped out the function on the floor, ask them to write the equations and graph the functions on paper.

Becoming the Functions (Individual Work)

Another thing I do with function work is to have the students each become functions. Any time we learn about a new function, I have students stand up and show me what the function looks like. Most students will automatically use their arms to map out the functions, but if a student draws the function in the air with their finger in front of them I will ask them to switch to using their arms. Drawing the function in space in front of your body is taking on an observer viewpoint, with the students are seeing the function in front of them. I want them to become involved with the function, and take on the viewpoint of the function itself.

It's a good way to wake up the students at the beginning of a class, or any time there is a need for some activity. As we learn more functions, we make a progression of the movements associated with each function:

$y = x$	arms are raised in a diagonal line
$y = x^2$	arms are raised up, in a U-shape
$y = \lvert x \rvert$	arms are raised up, in a V-shape
$y = x^3$	one arm is up, the other is down, with a a slight curve to each
$x = y^2$	a sideways U-shape
$y = \sqrt{x}$	only one arm is used, stretching up and out to the side
$y = \frac{1}{x}$	is challenging, the students may need to work with a partner

Make any of these functions negative, and the arms flip upside down. We practice moving up/down and left/right, and add in some expansions and compressions at times. I refer to the functions listed above as parent functions, and in Precalculus 12 we run through these functions with our arms regularly at the beginning of the semester in order to become familiar with how they work and with how they are related to one another.

Operations on Functions (Group Work)

Addition/Subtraction of Functions

Group the students into two functions and have them practice finding their shape, one group at a time. Then, have both groups find their shape on the grid at the same time. There should be two people for every input value (one from each function). Have the students who share the same input add their outputs and find their new position on the grid. Once all the students are sorted out, can someone draw a picture of the new function?

For example, have one group of students form the function $y = x$ and a second group form the function $y = \frac{1}{x}$. Can they map out their individual functions and then work together to determine what the graph of $y = x + \frac{1}{x}$ would look like?

Square Root of Functions

Have the students map out the line $y = x$ on a grid and then take the square root of the outputs.

They will need to determine what happens to those people who had negative outputs, which leads to a good discussion of domain. Work through a series of quadratic functions, mapping out the parabola on the floor and then taking the square root of the outputs. Ask the students to come up with equations and graphs on paper to illustrate what's happening when we take the square root of a function, based on what they do on the floor.