Kitchen science explores cone formation.

A Vignette promoting STEM experiments.

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Many mathematics advocates recall the *Mathematical Games* column that Martin Gardiner wrote in Scientific American for many years. However, few, if any of us, are old enough to have been around when the column began in the fifties. This means that a wealth of ideas is available for anyone who takes the time to scour the older issues. Inevitably a related column, *Amateur Scientist*, is discovered by anyone who searches the older issues. It ran for much longer but had a similar spirit of promoting experimentation by the readership.

While motivated by the spirit of the columns from the older Scientific American magazines, the questions used to develop this session arose from mathematics. The cone arises in volume and surface area problems and has a spot reserved in calculus with the classic problem of related rates of a leaking cone (how fast does the surface drop as the cone leaks at a steady rate). Where the cone arises in applications and why it is given attention in math class is an issue that does not receive much attention. How much ice cream fits in a cone does not do justice to the use of the cone and this motivated the idea of developing a STEM problem that highlights the connection between science and math. The practical importance of the cone means cone related formulae for volume and surface area are simply available tools, not the focus.

The mathematics curriculum in Ontario no longer includes conic sections. The ellipse, so useful for describing planetary motion, is not explicitly in the Ontario mathematics curriculum. The absence of conic sections, as a body of theory providing context to the cone, can lead some students to question the relevance of the cone. On an occasion when this issue was on my mind, an asphalt-recycling depot was making conical piles of crumbled asphalt. A photo was enough to explain the realistic question of how much material there was. However, this was also the STEM inspiration because it raises the question of how the shape of the conical pile might be different for different materials or circumstances. That is the fundamental question this vignette raises and explores.

Figure 1: Cone formation (Image from masoncontractors.org)

A key component to this vignette, as with any good science, is to raise the question in a manner conducive to the application of the scientific method and to promote hypothesis building. The session will aim to resolve some of the questions and hypotheses. In the same way many students have been asked to observe a burning candle, this session will ask participants to observe cone formation. The goal is not to resolve everything neatly; it is to demonstrate that a relatively simple activity can generate all the elements needed for STEM inquiry. It is hoped that the activity will spur further tinkering in classrooms and variations will promote further thinking.

As a starting point, imagine that material, such as salt, is placed in a funnel. At a given moment, the material is allowed to flow down the spout and land on the surface below where a cone will form. This setup has the advantage of rotational symmetry – something missing when the recycled asphalt was run off the end of a conveyor belt to form a pile. The asymmetric problem will not be addressed, but the hypothesis that it might lead to an elliptical cone demonstrates the spirit of opportunity to pursue this as a next step. The setup then needs to be examined to identify variables, build hypotheses and develop methods for testing the hypotheses. A diagram for the setup is shown in Figure 2. Consider, for instance, the following questions:

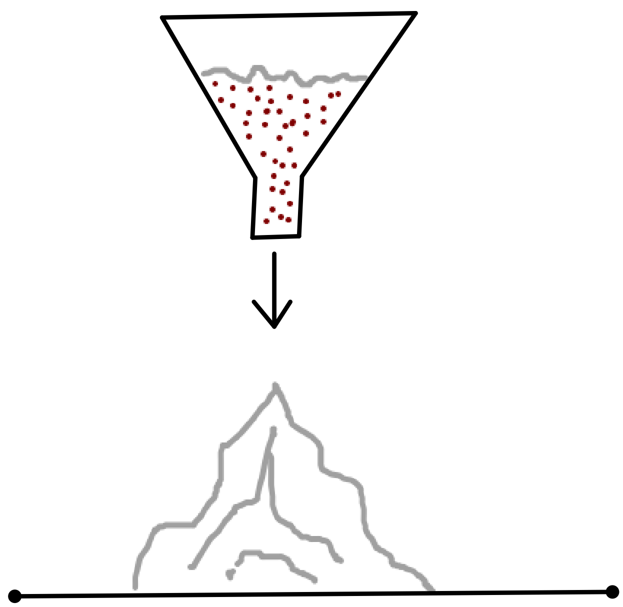

Figure 2: Diagram of experimental setup.

- ? Does the height of the funnel affect the shape of the conical pile? Raising the funnel would increase the time for gravity to accelerate material downward giving increased downward speed – would that impact where material landed?
- ? Do related rates make a connection between the funnel and the conical pile? The speed of material exiting the funnel varies due to the shape of the funnel; will this impact the formation of the lower conical pile? A related issue is whether the amount of material in the funnel at the start of the process has any impact.
- ? Is the conical pile genuinely conical? If it isn't, what shape is it and what impacts the difference? Is it reasonable to call it conical?
- ? How much does the choice of material impact the shape of the conical pile? Note that a pile of boxes makes a square based pyramid that has a different vertex angle than a tetrahedral pyramid made by stacking spheres. One might also wonder if materials

that flow more readily will lead to a different result than materials that do not flow as easily.

? Do properties of the surface, that the conical pile forms on, have any impact on the conical shape? (i.e. does friction on the landing surface affect the conical shape?)

With such a range of questions, the connections between the questions and the variables that exist in the setup need to be determined. In addition, some questions have clear connections and experiments can be developed rapidly, others require more time and debate may arise about how much a variable needs to be changed for experimental purposes. These are important considerations for a learning environment. Some variations, particularly extreme variations, may conflate variables and opportunities arise for discussion and debate.

Classroom organization is also a goal of this session. A single student, or pair of students, can develop an experiment and do many tests, with minimal variations to the setup. This improves experimental validity that facilitates drawing a conclusion. However, in mathematics classrooms there can be benefits to having many students contribute single measurements and then use the collective classroom results to develop patterns. For example, collective findings help build confidence for students who are unsure of themselves. A student, who errs, finds out before they have made too many measurements and committed too much time. In addition, collective findings facilitate opportunities to use data management skills in a meaningful way. While the scientific method may discourage the reduced validity of many contributing experimenters, it will allow for rapid testing of hypotheses. This approach to a STEM experiment will require a hypothesis is articulated and a method for testing it is communicated to everyone present. That element of communication and shared testing will be explored – it may demonstrate patterns more rapidly or be overwhelmed by experimental errors. In my experience, if the classroom is run effectively the former wins the day, while the latter leads to active student discussion about how to improve the explanation.