# This is math? Finding a way into mathematical thinking via puzzles & games (7-12)

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I've found that mathematical/logical puzzles and games appeal to a very wide range of people of all ages. In the classroom, they draw in students who are not normally interested in mathematics, they give students who may not excel at algorithmic manipulation a chance to show how well they can think logically to solve difficult problems, and they provide interesting challenges for students who find the curriculum straightforward. As there is no clear hierarchy among types of puzzles, there is no stigma involved in preferring one type over another.

While on sabbatical last year, I had the great joy of sharing mathematical/logical puzzles and games with over 4100 K-12 students and hundreds of teachers, either in the classroom or in workshops.

Some of the comments I often heard after visiting secondary classrooms:

*Is class over already?*
*Best math class ever!*
*When will you be back?*
*We can't stop now - I'm not done!*
*Where can I get more of these?*

Part of the attraction of our puzzle sessions was no doubt that the students thought they were getting out of math class. I usually did tell secondary students that they would in fact be doing all sorts of highly mathematical things, but that I hoped they'd forget that and just have fun.

When working with people of any age I have found it most effective to first display a completed puzzle and ask the group to figure out what the rules must be. Then we do at least one puzzle together; I insist that secondary students not only use mathematical language to make their suggestions but that they justify their reasoning. For example, "we know the bottom left box has to be black because if it were white there would be three in a row ", "7 goes in the third box from the right in second row from the top so it connects 6 and 8 diagonally", and "the 6-rectangle has to be two rows of three, because if it isn't it will block the 4-rectangle on the left". Thus we get some inductive reasoning, the use of precise language to describe spatial relationships, and the formulation of arguments that are clear to everyone. I'll play dumb if I have to; it usually takes only one or two vague descriptions or incomplete arguments for students to start to be very precise. At this point the teacher is usually grinning widely, no doubt hearing echoes of his or her own demands for clarity.

One of the striking outcomes of my classroom activities was that many students showed themselves as more mathematically able, engaged – and engageable – than their teachers had previously thought. I found this particularly meaningful at the secondary level, where many students have absorbed our society's attitude that it is not cool to enjoy doing math.

*Here are some of the puzzles that were highly successful with secondary school students – for more, come to the workshop for elementary students or check out my Dropbox.*

**Rectangles (Shikaku)**

Overheard about five minutes into a session with a group of rowdy – and originally skeptical – grade 7s: "Hey, I can *do* this!"

*The goal is to cover the grid completely with rectangles, with no gaps and no overlaps. Each number gives the size of the rectangle that encloses it; only one number appears in any rectangle.*

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | | | 4 | | | |
| | | 3 | | 3 | | |
| | | | | | | |
| | | 6 | 3 | 6 | | 5 |
| | 6 | | | | | |
| | | | 2 | | 2 | 2 |
| | | | 5 | | | |

I like this puzzle for several reasons. It combines numbers with geometry in a visceral way, making very clear the difference between prime and composite numbers. Because it involves shapes, the logic required seems easily accessible to more students than that required for some other puzzles. And, a minor point, it reinforces the idea that a square is indeed a rectangle.

Online-versions of this come in sizes up to 19 x 19. The bigger the puzzle, of course, the larger the numbers possible, which can make for good practice with factoring. It would be fun (and instructive) for students to make their own puzzles for each other.

Students who have mastered Rectangles might enjoy the further challenge of Filomino, which does not require the shapes to be rectangles, only that the squares involved connect along an edge.

**3-in-a-row**

*Each row and each column must have 3 Xs and 3 Os. We can never have 3 of the same type in a straight horizontal or vertical line.*

To introduce the game, I give each student two sets of coloured squares to place on their grid. The colours make it much easier to see patterns.

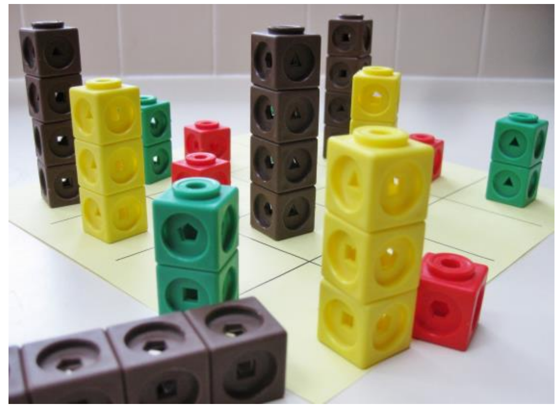
This seems as though it should be an easy puzzle, but as students often begin by just guessing, they quickly find out otherwise. It really requires discipline: which squares do we know *for sure*?

| | | | | | |
|---|---|---|---|---|---|
| | | | | | |
| X | | X | | | |
| | O | | O | O | |
| | | | | O | |
| | | | O | | |
| | X | | | | X |

Harder puzzles have fewer clues – but still only one solution – or a grid with more squares. This puzzle never gets extremely difficult, although it does require several types of logic, including parity arguements.

I have seen a version which states that we cannot get *four* of the same shading or symbol in a row, including diagonals. That puzzle grid was not a rectangle, which made it even more interesting.

## **Towers** (Skyscrapers, Utopia)



This is a variation of the Latin square. (See the K-6 vignette for a set of puzzles based on Latin squares.)

I really like this puzzle because it involves three-dimensional visualisation as well as logic. I have found it very helpful to introduce the puzzle using actual towers, colour-coded to make the patterns readily apparent. Eventually, keen students move on to using pencil and paper, or on-line versions, so that they can solve harder puzzles.

| | | | | | |
|---|---|---|---|---|---|
| | 3 | 2 | 2 | 1 | |
| 4 | | | | | 1 |
| 2 | | | | | 3 |
| 2 | | | | | 2 |
| 1 | | | | | 3 |
| | 1 | 2 | 2 | 3 | |

*There are four each of four heights of towers. Clues around the grid describe how many buildings are visible from that position. Each of the four heights must appear in each row and each column.*

Harder puzzles give clues for only some of the rows and columns. Some place a few numbers inside the grid, indicating the height of the a building in that position.

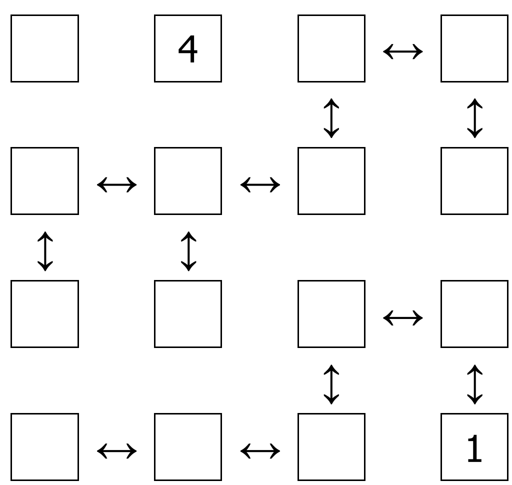
One can also make the puzzle harder by increasing the number of towers involved.

## **Neighbours**

Here is another variation of the Latin square.

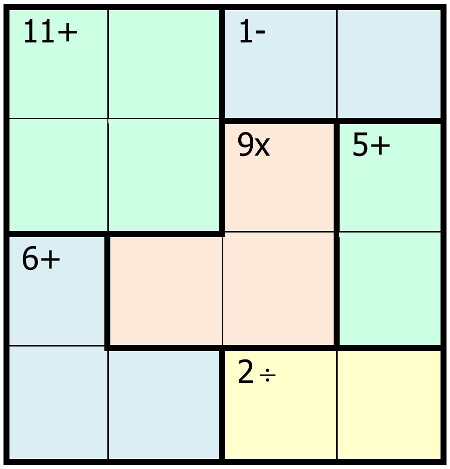
*Use the digits 1,2,3,4 once in each row and in each column. If there is a ↔ or a ↕ between two numbers, they are neighbours. For example, 2↔3. If there is **no** symbol between two numbers, they are **not** neighbours. For example, 1 and 3 are not neighbours, nor are 2 and 4.*

I particularly like the "anti-clue", which is unusual. When I introduce this puzzle to adults, there is usually a sense of "aha!" in the room, in recognition of the difference in kind of clue.




Neighbours is easily accessible but can get satisfyingly difficult – there are on-line versions up to 9x9.

**Kenken** (Mathdoku, Calcudoku)




One more variation of the Latin square. I find this puzzle far more satisfying than Sudoku because it requires basic arithmetic as well as logic. Bigger puzzles can get wonderfully challenging.

*Each numeral 1-4 must appear once only in each column and each row. The numbers in each heavy box ("cage") must combine to produce the target number in the top corner, using the mathematical operation specified. This can happen in any order, e.g. to get* $3-$ *you could have 1,4 or 4,1. A number can be repeated in a cage as long as it is not in the same row or column.*

All of these puzzles can be done by individuals or in groups, either competing or cooperating. In many classes I noticed the students taking great delight in finishing a puzzle before their teacher did, or in being able to explain something their teacher didn't quite get. It's not often that students see themselves and their teachers as intellectual equals engaged in learning the same thing.

There are many more puzzles suitable for secondary students. They may require modification for the introductory phase. The essential thing seems to be that the teacher finds the puzzle intrinsically interesting, beyond any educational value it might have.

Many of my puzzles are modified from on-line sites. Brainbashers and Simon Tatham's Portable Puzzle Collection are two excellent sources. Some are inspired by puzzles from assorted books, such as Moscovich's *1000 Playthinks of Art, Mathematics & Science* (Workman Publishing, 2001), and those spiral-bound puzzle books one finds in Coles/Indigo bookstores. A few are based on games one can buy commercially, such as SET.

Send me an e-mail ( susan.milner@ufv.ca ) if you would like access to my Dropbox , which contains these and many more puzzles and games at different levels which you can download and copy for your classes or use as templates to make more of your own. It also contains annotated lists of on-line resources and commercial games which I have tested on people of all ages.