Table of Contents

Foreword

Evaluating the health of a discipline at a national level seems a priori a difficult task, in view of the very many aspects that have to be taken into account.

The Natural Sciences and Engineering Research Council of Canada, in concert with bodies representing the Canadian mathematical community, decided to try and obtain a review of this nature from an international panel. Nine scientists, among whom seven mathematicians, were asked to form a committee in charge of fulfilling this request on the basis of their specialities, origins and responsibilities. Most of them are indeed involved in one way or another in structures regulating mathematical life in their respective countries.

Mathematical communities are facing challenges of different sorts in many countries, and Canada is no exception. Indeed, at the turn of the century, the structure of knowledge production is changing quickly, as are the social demands made on scientists and the need for a better scientific education for the general public. In the technical world in which we live, for better or for worse, understanding the core of science is no longer an issue that concerns only a small group of people involved in selective and protected professions. This fact has direct consequences on what is asked of mathematicians, on the way the mathematical profession is developing, and on the mechanisms by which it is evaluated and funded. It forces mathematicians the world around to project themselves into the future more than they ever had to do before.

The panel tried to keep all these aspects in mind, convinced that the health bulletin requested by NSERC authorities cannot be just a photograph of the present situation, but has to give consideration to the capacity of the Canadian mathematical community of responding to new challenges to come.

Relations with other sciences were very much on its agenda, since today no science can be thought of as an independent entity. Another issue which is not easily dealt with concerns jobs for students in Mathematics, since a proper balance has to be found between their entering industry or services (a healthy trend) and academic employment. Many countries, and several provinces in Canada, are currently implementing strict austerity budget measures with the effect of (sometimes brutally) blocking the replacement of personnel.

The dwindling of funds to support research, a widespread trend outside Japan, often leads to giving priority to short-term projects. In this respect, a discipline like Mathematics, even in its more applied component, is very often put in an awkward position because of the very conceptual nature of the field, resulting in difficulties in the use of programs too highly focused to pursue research of real value.

The panel was in constant interaction with the Canadian mathematical community and NSERC staff, as it should be, but enjoyed total freedom in organizing its work and expressing its views.

This report is the result of a 9-month process, a child delivery of some sort, with all the tensions that accompany such an event. It is the sincere hope of all panel members that it will prove useful, and help in a better understanding of the position and resources that the Canadian mathematical community has and can offer to Canadian society.

Let me conclude by taking the opportunity of these introductory words to thank Canadian colleagues, and in particular our convenor Richard Kane, for all their efforts to make the work of the committee feasible, and NSERC staff, mainly Danielle Ménard and Jean-Pierre Labelle, for their efficient support. More personally, I would also like to wholeheartedly thank all members of the panel for their full cooperation in this peculiar adventure.

Jean-Pierre Bourguignon
Chairman of the review committee

Review Committee

Jean-Pierre BOURGUIGNON, *chairman,*

Director, Institut des Hautes Études Scientifiques, Bures-sur-Yvette, France
President, European Mathematical Society

David BRILLINGER

Professor, Department of Statistics, University of California, Berkeley, USA

Mark GORESKY

Professor, Northeastern University, Boston, USA
Member, Institute for Advanced Study, Princeton, USA

David McLAUGHLIN

Director, Courant Institute of Mathematical Sciences, New York University, USA

Nicholas PIPPENGER

Professor, Department of Computer Science, University of British Columbia, Canada

Gilles PISIER

Professor, Université Pierre-et-Marie Curie, Paris, France
Professor, Department of Mathematics, Texas A & M, College Station, USA

Peter SARNAK

Chairman, Department of Mathematics, Princeton University, USA

Dan STROOCK

Chairman of the Division of Pure Mathematics, Department of Mathematics, Massachusetts Institute of Technology, Cambridge, USA

Margaret WRIGHT

Distinguished Member of the Technical Staff, Bell Laboratories, Lucent Technologies, Murray Hill, USA
President, Society for Industrial and Applied Mathematics

Executive Summary

1. Finding.– Canada has first class scientists in most major areas of Mathematics. As a whole, the Canadian mathematical community has achieved a high level of accomplishment and is well situated to attack the great challenges of current mathematical research. It is our opinion that NSERC's enlightened program of individual operating grants has been crucial in enabling the Canadian mathematical community to achieve this level.

Recommendation.– *The program of individual operating grants should continue to be the principal form of funding in the Mathematical Sciences and should be brought up to levels which put it on a par with that of other fields.*

2. Finding.– Involvement of Canadian mathematicians in teaching in a broad sense is remarkable. The widespread and outstanding quality of undergraduate training appears to be directly linked with the sense of participating in a global enterprise that Canadian mathematicians have developed over the years, thanks to the way various programs have been developed.

Recommendation.– *In view of possible adverse effects on domains of obvious national interest such as the quality of undergraduate training in basic sciences, NSERC is encouraged to proceed with extreme care when contemplating policies that could affect the perception of a common enterprise in the mathematical community.*

3. Finding.– As the production of the self-study document shows, the Canadian mathematical community has the tools to work as a group. Nevertheless, regional tensions threaten to handicap its development.

Recommendation.– *Although the application for a national network in Mathematics was recently not funded by the NSERC panel in charge of network applications, a structure of this nature should be used to channel energy towards joint ventures and cooperative activities.*

4. Finding.– The recent opening of two Institutes in Canada follows a worldwide trend. The coordinated operation of three institutes provides a stable structure which addresses the particular geographical situation of Canada.

Recommendation.– *NSERC should find a way to support all three existing institutes (the Centre de Recherches Mathématiques, the Fields Institute, and the Pacific Institute of Mathematical Sciences) by proposing a scheme of funding which encourages a national vision through collaboration and complementary activities.*

5. Finding.– The main resource on which mathematical research is based is qualified personnel. Many departments are threatened by major budget cuts which could dramatically affect their ability to attract creative students and young mathematicians.

Recommendation.– *Every effort should be made to smooth out the huge fluctuations in job availability throughout the country by offering appropriate programs and preventive measures.*

6. Finding.– The short term outlook for the academic job market is uncertain, and it is necessary to keep many talented people available for academic and industrial positions in the future. Some departments have set up programs to address these issues, and others are considering it.

Recommendation.– *There is a need to broaden the training of students within mathematics, incorporating requirements such as computer literacy. This is especially true for the doctoral programs at smaller universities.*

7. Finding.– Recent technical and structural measures resulting in termination or modification of NSERC programs have hurt the mathematical community although they were not intended to do so.

Recommendation.– *When designing or terminating programs, and even establishing rules or procedures, NSERC should carefully watch for differential effects on disciplines. Such effects may be minimized by maintaining maximal flexibility in NSERC programs and by using quality as the foremost criterion for funding.*

8. Finding.– We witnessed diverse efforts in the direction of "outreach" on the part of the mathematical community, some very promising, some more anecdotal. We did not get the impression that Canadian industry was as aggressively pursuing these opportunities as its counterparts in other countries were.

Recommendation.– *Outreach activities must be encouraged, but both NSERC and the mathematical community should be aware that the formats and contexts are likely to be unusual and unexpected.*

Part I : The Context

Special Features of Mathematics

Mathematics interacts with other areas in two ways: directly, when mathematicians create general-purpose methodologies and structures representing a wide class of problems, or seek to analyze, model, and solve particular problems arising in applications; and indirectly, when Mathematics developed solely for its intrinsic mathematical interest becomes relevant, sometimes unexpectedly, to a new problem. It is essential that *both* forms of interaction be recognized since they are of fundamental importance to Mathematics itself and to the rest of science and technology.

Mathematics is inextricably linked with science and technology for several reasons. Probably the most important is that abstraction —the underpinning of mathematics— has an unmatched capability for clarifying "the big picture" and identifying what is important.

It is a deeply held belief among mathematicians that their discipline provides a way of thinking and a set of techniques that can illuminate and sometimes solve problems from all areas of science, technology, and engineering. This view is not based simply on self-interest: Mathematics has a long, well-documented, and growing track record of major contributions to applications, and the pace of these connections is accelerating. For example, recent years have seen a spate of instances in which non-mathematicians have explicitly and publicly stated that Mathematics is essential to success, and even progress, in many areas of high priority for society.

Besides these facts, which are likely to shape the future organization of the mathematical enterprise, Mathematics has some specific needs and features. It is one of the most international of sciences, and mathematicians like to say that meeting colleagues belongs to their experimental apparatus. It is indeed true that confronting ideas is one of the most important parts of the process by which new concepts take shape. In recent years, there has been a worldwide trend towards establishing research centers with almost no permanent staff but running research programs devoted to special themes. This is a new dimension in the organization of mathematical activities.

Another crucial point has to do with the special relation that mathematicians entertain with written documents, since they do very often contain the final product of a piece of mathematical work. Hence, the passion that mathematicians all over the world devote to securing means for documentation, in particular books, paper and electronic journals.

Mathematics research has a strong social component: mathematicians typically think in terms of intuitions, pictures, and insights that are best communicated through conversations and discussions. For this reason, conferences and personal interactions are extremely important in Mathematics. On the other hand, mathematical progress depends predominantly on individual efforts, or on the work of very small groups. So "big science" is an inappropriate paradigm for most mathematical research. As we shall see later, NSERC's individual research grants program is extraordinarily well suited for the support of mathematical research.

Finally, a feature that singles out Mathematics is its pressing quest for unity, in spite of its many ramifications in many different areas. This has to do with the permanent internal reorganization that Mathematics is undergoing, by which domains that at some moment were considered foreign to one another later appear to have deep and inspiring links. This drive is not at all new, but has been playing a major role in the most recent developments of Mathematics.

The International Context

The points we have just made are not recognized in a uniform way by all the countries and communities questioned. It is indeed a fact that the XXth century has witnessed a fantastic development of Mathematics, and of many other sciences, but the success story that mathematical research has had in recent years remains in many circles a well kept secret.

This is all the more surprising when one realizes that Mathematics is very widely taught in secondary schools around the world, and that almost all universities host a mathematical department. This special relation of Mathematics to teaching has complex consequences. First, it ensures mathematicians living in the academic world that a good part of their students will consider taking teaching positions. (This is at least so in the many countries where teachers' training largely relies on disciplinary knowledge.) It may have played a role in the late recognition by mathematicians of the need to open up, after a period in which they were mainly busy with internal developments and not so eager to involve themselves in external activities. The rapid development of mathematical modelling and its main impact on many advanced technological projects has created a major market for mathematically trained students, hence the need for more diverse curricula.

The contribution of Mathematics to the societal life is not always given its due. One further reason may lie in the fact that the way in which Mathematics gets involved cannot in general be reduced to an automatic response to targeted programs, set up to achieve a short-term task.

All these facts underpin the (sometimes new) needs expressed by mathematicians, and call for special attention to offers made to support them.

The Canadian Context

Canada is a country which had a priori the natural background to host an active mathematical community, and it has done so for a long time. It is certainly appropriate to recall here that the highest distinction that a mathematician can receive bears the name of a Canadian mathematician, John Charles Fields. The renowned hospitality of Canada towards European immigrants also played a major role.

Some geographical facts are also shaping the internal mathematical life of Canada. It is indeed a country in which populated areas spread over a few thousand miles, and with a population of about one tenth that of its powerful neighbor, the United States, and one half that of France, a country that still keeps special relations with at least part of Canada. The high economic level of the country makes it the home of very advanced industrial companies, a priori a favorable point for the development of Mathematics.

The provincial structure is also part of the general picture. It determines a large part of the hiring pattern for academic positions, but it is to be noted that different provinces often pursue similar policies at different times and paces, introducing phasing effects in the life of the mathematical community.

We have to say that what prompted the constitution of a panel to review the situation of Mathematics in Canada is the outcome of the last NSERC Reallocation Process in which Mathematics was ranked very low among other disciplines, and the consequences that resulted from this evaluation in terms of allocation of funds. This event generated a huge reaction from the mathematical community. What is remarkable is that the internal effects have been concentrated towards finding a better organization, an increased level of internal communication, and a tremendous drive for new global initiatives from the community as a whole. This happened at a time when the overall job market in North America for trained scientists, and in particular mathematicians, was entering a very depressed period. Even more than that, many mathematicians witnessed the shrinking of some important departments through cuts in the budget devoted to higher education.

On top of this blow, which has both an intellectual side and a practical one, various technical measures taken by NSERC (such as the termination of some programs, and among them the one designed to support travel to conferences) gave mathematicians the impression that their specific needs were not recognized. Even if the support to the newly born Fields Institute did show that important initiatives were not ignored, and were provided with the basic means that their development required, the measures aroused a feeling of frustration and, for some mathematicians, low morale, although a strong national campaign was launched. This resulted in the request made by the Canadian mathematical community to NSERC authorities for the constitution of an international panel. It was given the mission to evaluate globally the community's performances, and its capability of responding to future challenges. The NSERC President, Dr. Brzustowski, played an active role in launching this process, and was kept personally informed of its progress.

Methodology

This panel was constituted after substantial consultation between representatives of the Canadian mathematical community and NSERC officials, and the chairman. Great care has been taken to ensure that it can cover a wide area of Mathematics, that panel members have sufficient knowledge of the Canadian scene, and also sufficient experience in dealing with global issues concerning the organization of mathematical communities. Two Canadian scientists, specialists of areas close to Mathematics, joined the panel.

An important role was given to the convenor, a Canadian mathematician, with the formidable task of centralizing the preparation of all necessary material, in particular a Self-Study document presenting globally the situation of Canadian Mathematics. Putting together such a comprehensive survey of this sort in about four months from a community of approximately 1000 members is already a sign of an exceptional cohesion, and capacity to respond to challenges.

Other documents, also made available to panel members in the late Summer, were related to the work of the Grant Selection Committees, to reviews of the existing institutes or to applications made to the Network Committee such as the National Network Initiative which includes the Pacific Institute for Mathematical Sciences. The Committee also had access to statements prepared by the Mathematical Liaison Committee.

After an initial meeting held on October 18 at the Fields Institute in Toronto, where panel members could get to know each other, and to define the method to be used, the panel split for site visits into two groups. One was to meet Canadian mathematicians from the West with stops at Edmonton and Vancouver. After meeting mathematicians working in Ontario, the other one went East to Halifax, to cover the Maritime Provinces, and to Montréal to cover Québec. All these meetings were thoroughly organized locally by correspondents who had extensive preliminary contacts with the convenor. Presentation of research by themes, of outreach activities and meetings with Ph.D. students or post-docs were organized at each stop. The level of the presentation was at all places remarkable, and a large cross-section of regional activities was presented. In spite of a very tight schedule, the visits turned out to be very informative, with the result of making some panel members change their minds on some of the activities that were presented orally to them.

The second meeting was held in London on December 7-8 on the occasion of a meeting of the Canadian Mathematical Society. This was an opportunity to meet chairs of departments who were holding their annual meeting on this occasion. At this second meeting, it was possible for panel members to confront the scientific findings made during the site visits, to discuss the structure of the final report, and to identify the key issues to be stressed. Dr. Brzustowski accepted an invitation to meet the panel there, and also to attend a presentation made to the panel by the Mathematics Liaison Committee. Both meetings were very successful and informative. They were attended by the convenor and by NSERC staff.

Later in January, the panel did, as was suggested by NSERC officers, express its views about the peculiarities of the National Network application in a letter to the ACORN panel in charge of examining network applications. When the decision to reject this application was made known, putting the PIms project at very high risk, the panel felt it was its duty to express its views on this special project, and the possible consequences of letting such an opportunity pass. That is why it sent a letter to Dr. Brzustowski calling attention to this problem.

The report was completed in the months of March and April through consultation between panel members.

Report Structure

It appeared to the panel that, after the presentation in Part I of the general context and the method used, the report should begin by the scientific facts concerning Mathematics in Canada. This is why Part II is devoted to scientific findings.

In recent years, one important issue concerning the mathematical community has been its internal organization, and the funding structure in which it has to live. This is the topic addressed in Part III. It of course includes the special position of institutes in this landscape, and also the outreach and industrial partnerships.

The terms of reference also require that the panel take a prospective view, and look firmly into the Future. This is taken up in Part IV.

An Executive Summary, gathering Findings and Recommendations, has been placed for convenience right after the Foreword. Extreme caution should be used in isolating it from the general arguments given in the report itself.

For the sake of completeness an annex presenting the terms of reference of the review committee has been added.

Part II : Scientific Findings

General Comments

It is traditional to try and divide Mathematics into subdisciplines. Names such as Algebra, Geometry and Analysis have certainly spread outside the circle of mathematicians. Such a division, or even a more refined one, if it helps to put mathematicians into groups of individuals that share most of the time a common language and common techniques, does not give the right image of the discipline. In particular, it does not do justice to its dynamics which moves the internal boundaries between domains at an ever accelerating pace, and brings in contact sub-disciplines which previously had not much in common.

When speaking of the activities of about one thousand mathematicians in a few pages, there is no room to give an adequate catalog of all scientific achievements that would normally deserve to have been highlighted, of all bridges that have been built and of all interesting viewpoints that have been developed.

In this report, the panel decided to focus its attention on world class achievements, exceptionally high level groups and topics which in Canada have recently had special developments. This view at a distance will of course leave out all details under a certain scale. Another limitation has to do with the size of the panel itself, but we hope to have filled in our worst gaps. Since World War II, Mathematics has grown remarkably in Canada as in many countries, and the overall impression is that in Canada there is not a single subdomain which could be considered as frozen in isolation. Canadian mathematicians are participating at a high level in the thriving enterprise of mathematical research.

In what follows we have tried to organize the presentation under disciplinary headings, although we are aware, as already pointed out, of the somewhat arbitrary character of this way of cutting up mathematical activity.

Foundations

The advent of computers has given Logic new significance and potential for application that complement its traditional foundational role within mathematics. There is much scope for new mathematical impact in this development, which emphasizes aspects such as Finite Model Theory and Modal Logics that have until recently often been viewed as peripheral. Much of this work is taking place in Computer Science and even Philosophy departments, but the knowledge and skills of mathematicians put them in a position to make much greater contributions here than they have in the past.

Set Theory and Model Theory have in recent times had remarkable success in resolving the foundational status of many mathematical questions in areas such as analysis and topology. A substantial group of researchers in Ontario have made important contributions to this development.

Canada also has researchers who have played leading roles in the development of Recursion Theory, Universal Algebra and Category Theory, and in the case of Category Theory there are significant groups in Montréal and the Atlantic Provinces that are pursuing its applications both within Mathematics and outside.

Discrete Mathematics and Combinatorics

The areas of Discrete Mathematics and Combinatorics have known an explosive growth throughout the world during this century, and especially in the period since World War II. At the beginning of the century, the term "discrete Mathematics" was not in use, and Combinatorics was a relatively isolated area that was tied to the main body of Mathematics primarily through the fact that combinatorial processes underlie the determination of the coefficients in many series. Since World War II, however, Combinatorics has grown both in breadth and in depth, driven in large part by its potential for applications in Communications, Computer Science (both of which made a transition from analog/continuous to digital/discrete during this period) and in Operations Research (among other areas). This growth has been accompanied by a great strengthening of the ties between Combinatorics and other areas of Mathematics, creating substantial bodies of work at these interfaces. This has led to the recognition of a broader area of Discrete Mathematics, which includes Combinatorics but also overlaps Algebra (especially in the theory of finite fields, permutation groups, and number theory). The range of mathematical techniques in discrete Mathematics has significantly expanded; although *ad hoc* methods were once prevalent, it is now not uncommon to find Discrete Mathematics drawing on Probability Theory, Harmonic Analysis or Algebraic Topology.

There are active groups throughout Canada in Discrete Mathematics, Graph Theory, and Combinatorial Optimization, including widely respected and recognized international figures. Many of these researchers are not affiliated with Mathematics departments, and hence may not be labeled as "mathematicians". It is a characteristic of Canadian research in Discrete Mathematics and Combinatorics that, although the first name one might think of in any particular area might not be Canadian, there are strong Canadian groups in virtually every area of the field. This puts Canada in a good position to explore potential opportunities for applications, which in their incipient stage usually expand from a closely related and already established area of Mathematics.

Algebra

This area, dealing with structures and algorithms, has very old roots. It is exemplary of the formidable way in which Mathematics has been expanding in this century. Although the XIXth century witnessed already remarkable progress in Algebra with the emergence of all the major concepts that now structure the subdiscipline, such as groups, rings, fields, ideals, it is in this century that Algebra has connected successfully with other areas of Mathematics because algebraic structures could be used to study many different fields, e.g., Algebraic Topology and Algebraic Geometry. Both of these domains are dominated by the power of tools borrowed from Algebra, and remain very close to its spirit.

Algebra and algebraic groups are strong, international, and well represented in Canada. This group includes several senior leaders who, through their research and exposition, have influenced the shape and direction of the modern theory of algebraic groups.

An area which of course is directly affiliated with Algebra is the representation theory of groups and algebras, i.e., the study of groups and algebras of transformations of structured spaces (vector spaces, metric spaces, etc.). This theory was vastly extended in the second part of the century, in particular to accommodate the needs of theoretical physicists challenged by questions coming from quantum mechanics. A Canadian mathematician, Robert Moody, was one of the founders of this extended and fruitful theory, now called Kac-Moody algebras. Expertise in this domain has been maintained.

Number Theory

This field does not need any introduction since the concept of number speaks for itself, even if mathematicians like it a bit more sophisticated than the layperson. Understanding the structure of the set of prime numbers, or the irrational or transcendent natures of some universal numbers such as π or e has forced them to unravel deep mathematical facts. Number theory is a domain in which many of the problems left unsolved by previous generations are deep and challenging. (Think of the Fermat's Last Theorem.)

Number Theory is perhaps the leading field in Canadian Mathematics. As a group, it is competitive with the top five or so number theory groups anywhere in the world. Canada has succeeded in hiring some very bright young number theorists, and the whole community is very up to date. At Toronto and McGill there is more than a critical mass of activity in this field.

James Arthur is the world leader in the modern theory of automorphic forms. The trace formula techniques which he developed over a fifteen year period have become essential tools in representation theory, automorphic forms, and Langlands' program. (Note that Langlands himself is a Canadian, presently a permanent member at the Institute for Advanced Study.)

John Friedlander is a world leader in prime number theory and L-functions. In joint work with Iwaniec and others, he resolved many long standing conjectures in classical number theory (such as the infinitude of primes of the form $x^2 + y^4$). Problems which number theorists have grappled with for over one hundred years have been solved using their machinery.

Other Canadian number theorists have a strong international presence with striking results to their credit.

Geometry

Originally, Geometry means "measuring the earth". It was later understood as the study of forms. After it became clear that Euclidean space was not the only form of space relevant for Mathematics, and more generally for Science, the notion of geometry could be enlarged to the creation and analysis of new forms of spaces, as was envisioned by Riemann. In the second part of the XIXth century, it was identified by Klein, Helmholtz and Clifford with the general study of properties invariant under a group of transformations. One of the main driving forces behind the evolution of Geometry in this century has been the quest for global properties (as opposed to local properties, which were the main focus of research in the XIXth century).

Differential Geometry is divided today into several branches. We stick here to the Riemannian and symplectic ones, forced as we are to be schematic. The first generalizes the study of surfaces in Euclidean space, and has been very much stimulated by the theory of General Relativity. In Einstein theory, the vacuum itself has an interesting structure, the so-called Einstein metrics. Canada has some very good specialists in this area.

Symplectic Geometry is the outgrowth of the rich internal structure of Analytic Mechanics, as developed by Lagrange, Hamilton, Jacobi, Poincaré and Emmy Noether. After a sleepy period in the first half of the XXth century, the theory has come to life again, and presently this is an area of intense current excitement the world over. Canada has been successful in attracting a very impressive group of young mathematicians in the Montréal area who have resolved several well known conjectures in symplectic geometry and topology. The members of this group are much sought after and special efforts will be needed to keep them in Canada.

The study of some properties of algebraic spaces by transcendental methods relies also on this type of Geometry, in a complex setting. This connects with the study of Topology and of Mathematical Physics because of the recent strong interest shown by theoretical physicists for moduli spaces of geometric structures, a natural object of study for geometers. Here again, several Canadian mathematicians are contributing at the highest level on this subject.

Topology and Dynamical Systems

This section may be the most problematic to present since first of all the word Topology has two meanings, and secondly because the material to be covered under this heading is in such close contact with other areas that one is tempted to mention it elsewhere. In fact, this very dynamic area (no joke intended) is exemplary, on its scale, of what is happening in Mathematics as a whole, namely, multiplication of the interactions with other areas (inside and outside Mathematics), borrowing of techniques from many different subdisciplines, stimulation by the applied sectors (Engineering, Biology and Ecology just to name a few), possibility of a more experimental approach thanks to the tremendous increase in computing power.

The word “Topology” is used in Mathematics to name a structure that enables one to speak of proximity and convergence in the most flexible way, e.g., without having to rely on a notion of distance. This gave rise to a science of shapes in this malleable sense. This is the way mathematicians see Topology as a branch of Mathematics. It has some very basic and challenging open problems. To tackle them, algebraic, geometric and more recently analytic tools have been mobilized, hence for specialists of this subject the need for contacts with these other broad areas of Mathematics.

This viewpoint gained much weight after it was recognized that it shed light on Poincaré’s call for qualitative methods in the study of Celestial Mechanics. He had indeed recognized that the traditional analytic approach could not be successful. This lead to the birth of the theory of Dynamical Systems, as a broadening of the theory of ordinary differential equations, to include both a global qualitative point of view and possibly discrete versions. The new theory immediately retained the attention of many scientists, as one from which they could get important informations concerning their own fields. Notions such as quasi-periodic, chaotic, or stable behaviours are now widely used. Sometimes mathematicians feel that they are not used with the rigour that would be necessary to draw solid conclusions. Canada has an active school in this area, with interesting connections to Control Theory and Differential Geometry for example.

To come back to Topology as a mathematical discipline, it is also well represented in several departments in Canada. In particular, there is an active, highly visible group of well established mathematicians in Geometric Topology and Homotopy Theory, several of whom are internationally recognized.

Analysis

On our tour, Analysis appears as the last of the classical branches of Mathematics. This has nothing to do with any qualitative ranking since, in this century, it has been undergoing the same radical transformation as the others. The master word for a good part of Analysis is "to estimate" (think of limits, series and integrals !), but the framework has been enlarged to the study of spaces of functions, bringing into Functional Analysis a geometric viewpoint, exemplified by the role played by Hilbert spaces as infinite dimensional generalizations of Euclidean spaces, or by Banach spaces, with their more subtle internal geometric structures. This approach has been highly successful in the resolution of Partial Differential Equations, an area which, because of its many ramifications, appears as a separate item in our presentation. Nevertheless, there is still room for very fine analytical developments (some people like to stress its deepness by calling it "Hard" Analysis), such as the ones that can be found in Harmonic Analysis. This is an area in which Canada has a definite tradition that has been kept alive. Indeed, there are in Canada many first rate mathematicians in areas working on Harmonic Analysis, Potential Theory, or the Geometry of Banach spaces,

Many areas of Analysis have grown enough to have their independent lives. One of them is undoubtedly the study of Operator Algebras. This field is currently flourishing in Canada as is Operator Theory (the study of one operator for itself). By an operator algebra one usually means a C^*-algebra (hence a self-adjoint algebra formed of bounded operators acting on a Hilbert space). In the last 25 years, their theory has enjoyed spectacular advances on the world scene with many interactions involving other areas. The original motivation for studying operator algebras goes very far back to the foundations of Quantum Mechanics (where numerical values of measurements are replaced by the spectrum of self-adjoint operators) and especially to von Neumann's pioneering work on the rings of operators (now called "von Neumann algebras").

In Canada at the moment, the leading figure is undeniably George Elliott, who is the architect of an ambitious program of classification of C^*-algebras by K-theoretic invariants. This program (referred to, worldwide, as the "Elliott program") has advanced incredibly in the last decade, mainly because of Elliott's own outstanding contributions, but also because of his ability to recruit a number of first rate talents, and communicate his enthusiasm. This was particularly visible during the special year (94/95) organized by him at the Fields Institute, which was a tremendous success.

In the last 20 years, the study of non self-adjoint algebras (which necessitates quite different methods) emerged as a somewhat new "subfield" which, while much smaller at the world level, is well represented and very strong in Canada.

Partial Differential Equations

One way or another, partial differential equations (PDEs) crop up not only in most branches of Mathematics but also in essentially all aspects of Theoretical Physics. Here are a few examples which are chosen to demonstrate just how pervasive the field really is :

(a) *Geometry.* Ever since the work of Weyl, Bochner, and Hodge, it has become apparent that major advances in Differential Geometry involve the analysis of a PDE, e.g., Patodi's heat flow approach to the Atiyah–Singer Index Theorem and the construction by S.-T. Yau of what have been called ever since *Calabi–Yau* manifolds

(b) *Number Theory.* One of the most hopeful formulations of the renowned Riemann hypothesis is given in terms of the Laplace operator on the Poincaré upper half-space.

(b) *Probability.* PDEs provide the single most powerful tool with which to deal with quantities which arise in both the practice and the theory of probability

(c) *Combinatorics.* The time-honored technique of generating functions allows combinatorialists to convert seemingly intractable counting problems into occasionally more tractable questions about PDEs.

(d) *Physics.* Newton, Boltzman, Maxwell, Einstein, Schrödinger all expressed the basic equations of their theories in terms of PDEs.

In view of its breadth, one should not be surprised that only some aspects of the field are represented in the Canadian Mathematics community. Indeed, it could hardly be otherwise. Thus, the fact that in their own *Compendium of Canadian Mathematicians*, the topic PDEs is listed as an addendum to Mathematical Physics is both understandable and an accurate assessment of the field's strongest practitioners in Canada.

Victor Ivrii, in the spectral theory of hyperbolic PDEs, and I.M. Sigal, in the theory of the Schrödinger equation, are among the top three or four figures (worldwide) in their specialties. In addition, Canada should take pride in the presence of several other gifted specialists in the theory of linear PDEs. Of particular note are some of the recent junior appointments which have been made in both Québec and Ontario. If one assumes that Canada can satisfy the expectations of these younger people, then prospects for Canada's future presence in this area is bright.

Because the modern theory of non-linear PDEs is still in its infancy, there is less unanimity about what constitutes *great* work in the field. Nonetheless, Canada possesses reasonable representation in several of the directions which look most promising in this field. Perhaps the most intriguing work is taking place in the western provinces, where there are several applications oriented groups combining numerical and theoretical methods to attack equations which arise in a variety of applications. The practical success of their efforts is reflected in the support which several members of these groups have garnered from non-governmental sources.

Mathematical Physics

Mathematical Physics has a long tradition of stimulating research in Mathematics, and in this part of the Report, we have already come across several instances where motivations coming from Theoretical Physics played an important role in the emergence of new mathematical concepts. Modern Physics, both quantum and relativistic, calls for some mathematical sophistication. The new trend, that has had an exceptional impact on the most recent developments of Mathematics in the last ten to fifteen years, finds its origin in new insights on some mathematical objects that could be drawn from a "quantum" vision of some objects or methods such as Quantum Field Theory. The impact has been as strong on pure Mathematics (Algebraic Geometry, Global Analysis, Topology, Knot Theory) as on more applied areas (Probability Theory, Hydrodynamics, Dynamical Systems).

A strong tradition of Mathematical Physics exists in various parts of Canada, and notably in the Montréal area. This was one of the driving forces behind the foundation of the Centre de Recherches Mathématiques there. In some of the more analytical aspects, Joel Feldman in Vancouver has gained a world position, but he is not at all isolated, and the Western universities have developed remarkable teams in this areas of research too.

One aspect that the panel has difficulty in estimating is the frequency of the day-to-day working relations with physicists. This is very important to make sure that the problems tackled are indeed drawn from present day Physics, even if one has to take into account the different time constants that Mathematics and Physics have. This interaction has been and continues to be one of the most active and fruitful stimulations for mathematical research. Canada seems well equipped to take advantage of this exceptional moment of convergence.

Probability and Statistics

Probability is an important field of Mathematics, although its development occurred mostly in this century. It stands as a mathematical subject on its own, particularly its foundational and theoretical material. At the same time it lies at the interface of Mathematics with other subjects (such as Statistics, Physics, Chemistry) and with the natural world. Probability is almost directly driven by new developments in Mathematics (e.g., probabilists seek stochastic variants of new mathematical structures), yet at the same time it influences the development of Mathematics itself by providing tools useful in the study of mathematical concepts (such as solutions of differential equations). All in all, strength in Probability is crucial for strength in Mathematics. Also because the techniques of the subject handle uncertainty and measurement error, knowledge of Probability seems crucial for professionals in any analytical field.

Canada is fortunate to be strong in Probability. The University of British Columbia has a number of fine probabilists. Ontario also has recognized leaders in their individual fields. In terms of world wide recognition one can point to the fact that five of these probabilists have presented invited lectures at the International Congress of Mathematicians, which occurs every fourth year and is one of the undisputed signs of world visibility. Outside of British Columbia and Ontario one notices individuals carrying out novel work with stochastic processes, particularly applied to signal processing. It can be noted in this connection that there are Canadian researchers in other fields, e.g., Physics and Engineering, also making basic contributions to the subject matter of Probability.

Most of the individuals highlighted above are young. It can be anticipated that with basic support and nurturing as their work continues some of them will be recognized as stellar researchers on the international stage.

Applications of Mathematics

Throughout the world, the implication of Mathematics in applications continues to be increasingly important in a growing set of contexts, and this trend is likely to accelerate as modeling and computing become inextricable components of Science and Engineering. There are very different aspects to this phenomenon. Some require the creation of new concepts in Mathematics, some stimulate the development of new techniques, some bring into Mathematics new points of view. Familiarity with computing, which is almost systematically required when dealing with applications, also has an impact on the way mathematicians behave in their own field when doing research, and when teaching. This widened impact of applications involving various other fields of knowledge also pushes mathematicians to be less illiterate in other sciences, an attitude which was unfortunately fashionable among mathematicians at one time.

It is very difficult in such a short document properly to present the variety of skills which are required when dealing with problems coming from areas so different. Each one of them has its scale, its modus operandi, its schemes of validation, its time frame. Listing the areas of applications can give a false impression of homogeneity in the demand made on mathematicians when addressing the questions posed. This would be a tragic misconception. As a result, although the segment of the mathematical community involved in such activities has grown in the last ten to twenty years, there is still much room for further involvement in very diverse directions.

Canadian applied mathematicians are involved in many core areas ; a community of researchers belonging to academic departments in Mathematics, Computer Science, Physics, and Engineering; a widening focus on interdisciplinary work; and new programs and applications.

Overall, across Canada, research in applied and computational areas of Mathematics is strong and healthy. Canadian researchers enjoy high international repute in Numerical Analysis and Scientific Computing. Noteworthy contributions have been and continue to be made in a variety of core applied areas, including Fluid Mechanics, finite element analysis, Dynamical Systems, numerical solution of PDEs and boundary value problems, Control, and Optimization Theory. Canadian applied mathematicians have produced numerical and symbolic software packages used around the world —AUTO, COLSYS, MAPLE, and SPARSPAK, to name only a few.

Canadian researchers also maintain a high scientific profile in several key application areas. Mathematical Biology, including Physiology and Ecology, is emphasized and well regarded country-wide. Applied dynamical systems theory has significantly affected work in Physiology, and Mathematical Ecology has directly influenced the fishing and forestry industries in Canada. Other application areas of special note are Geophysics, weather and climate studies ; Finance ; nonlinear waves and Fluid Mechanics ; Materials Science ; and Microelectronics.

Academic applied mathematicians in Canada frequently lead or participate in research groups and academic programs that are explicitly interdisciplinary. Such research groups, often cross-departmental, are valuable in attracting young faculty, postdocs, and graduate students who wish to combine mathematical research with other scientific areas. For example, the program in applied Mathematics at UBC has provided graduate training in interdisciplinary science and associated Mathematics for 25 years ; applied groups and programs in Toronto are affiliated with Mathematical Physics, Finance, Probability, and Computer Science; applied mathematicians spanning several universities in Montréal have close ties with Mechanical Engineering, Biology, Medicine, Physics, Structural Engineering, and Materials research.

Our committee was struck by the emergence of lively and innovative programs connected with applications of Mathematics. For example, financial Mathematics has inspired the establishment of several degree programs connected with, and in some cases supported by, industry ; the Center for Experimental and Constructive Mathematics at Simon Fraser University is intended to explore and create links between Mathematics and modern computation. The vitality of these and many other programs provides a clear indication of the health of the community of Canadian applied mathematicians.

Part III : Organization and Funding Structure

General Facts and Trends Concerning the Organization

For a long time, mathematical research was conducted by university professors who were in contact with a small group of colleagues interested in the same questions. The cost was minimal, and the connection with other human activities was achieved through the training of students, or individual consulting. This era is over for at least three concurrent reasons :

* The pace at which research is progressing and, it must be recognized, pressure to publish research papers have induced mathematicians to work in teams whose members may not be located in a single place and communicate via electronic and other modern means. This requires more organization, more exchange to keep abreast of new developments, and greater willingness to look for opportunities to learn completely new material from neighbouring or even remote fields.
* A number of branches of Mathematics make heavy use of computers (either workstations or machines accessible through the network), hence mathematicians have to think about equipment, to worry about its maintenance and its obsolescence, and to master programming of one sort or another.
* The multidisciplinary aspects of research in which more and more mathematicians are involved force them to care more about contacts with specialists of other fields, or ones connected to industry or to services.

Most universities in Canada have mathematics departments which are rather comprehensive, incorporating pure and applied mathematicians, mathematical physicists, sometimes statisticians, and occasionally computer scientists. This of course has to do with university sizes. Canada has no structure hosting permanent positions devoted exclusively to research, such as the Institute for Advanced Study in the US (on a very small scale, one must say), or the Centre National de la Recherche Scientifique in France. On the basis of the data provided, it is difficult for the panel to be very precise about either the number of mathematicians employed as such in industry, and the research infrastructure in which those mathematicians are working. The impression it had from what it heard was that Canadian industry (or Canadian branches of international companies) did not seem to have a very aggressive policy in this respect.

One of the consequences of the organization just described is the large part which teaching has in the careers of most Canadian mathematicians. This situation is not unique to Canada, but the panel members got a sense that Canadian mathematicians have a stronger than usual commitment to their teaching duties. No doubt, Canadian students are the beneficiaries of this commitment, which ensures that students will be exposed to new results, new techniques, and new trends. At the same time, there is a risk that this commitment will be abused by the imposition of heavy teaching loads which leave no time for research.

At the present time, Canada's success in teaching mathematics to undergraduates is probably better than that of any other country in the Western Hemisphere. What is particularly impressive is that this success is so widespread. Without question, a major factor is the feeling among mathematicians, even at otherwise isolated institutions, that they are all part of the Canadian mathematical enterprise. Since this sense of community can be preserved at modest expense but, once lost, could be regained only at a very high price, it strikes this panel as poor fiscal judgement for Canada to take, even inadvertently, measures that would jeopardize it.

In spite of the organizational changes alluded to above, mathematical research requires ready access to not only the traditional bibliographical data found in libraries but also to the sort of non-traditional data made available via electronic networks. In addition, mathematicians still require personal contact with their peers, either through scientific visits (including post-docs), participation in conferences and workshops, or attendance of special programs. As a consequence, even though it remains far less expensive than most experimental sciences, the costs of maintaining a thriving mathematics community are no longer negligible.

Institutes

As mentioned earlier, there has been a worldwide trend in recent years to open institutes with the specific tasks of providing an efficient environment for short term visits, either to enjoy undisturbed freedom to concentrate on research projects, or to participate in well focused workshops, or conferences. Opinions among mathematicians vary on the soundness of this general policy : some fear that university departments be slowly deprived of their responsibility of conducting research activities, others worry about the possible uniformizing effects that meeting the same experts the world around could cause. Positive sides are nevertheless numerous : the possibility of being involved in more ambitious research projects because of the concentration that such an environment offers ; the possibility of reaching a critical mass on a given subject ; the possibility for a researcher, who otherwise might be isolated in his or her institution, to be fully integrated in a very competitive team ; etc.

Canada has two such institutes working at full capacity : the Centre de Recherches Mathématiques located in Montréal, and the Fields Institute located in Toronto. Another structure, which can be assimilated to an Institute, has begun its activity in the West, the Pacific Institute for Mathematical Sciences, on which we comment as a new project.

The Centre de Recherches Mathématiques

The Centre de Recherches Mathématiques was created almost 30 years ago, and started to play a national role more than 10 years ago. It is located in a brand new building on the campus of the Université de Montréal of which it is a part. It involves a number of research institutions from Montréal, but it is also supported by NSERC and FCAR, the research agency of Québec. It has already established a tradition of visitor's programs, post-doctoral fellowships, and monographs that are widely distributed. The Aisenstadt chair has attracted world class mathematicians and the Aisenstadt prize distinguished very promising young Canadian mathematicians. Its scope encompasses the mathematical sciences in a broad sense with a slant towards Mathematical Physics. Recently run yearly programs have covered a broad spectrum of topics, the more recent ones focused on Dynamical Systems, Geometry and

Topology and on Applied Analysis and Numerics. They seem to have been especially successful.

The presence of mathematicians from Montréal has been ensured by a mechanism of reduced teaching loads distributed to faculty members. The impact of such possibilities is certainly not to be forgotten when it comes to evaluating the attractiveness of Montréal as a mathematical center. The implication of mathematicians from other regions in Canada has been ensured by programs like, for example, the summer schools in Banff.

Great efforts have been made in recent years to conduct successful outreach activities. Special partnerships have been established to ensure collaboration with non-mathematical departments of some of the universities of the Montréal area (Physics, Biology), and some of the institutes in Québec oriented towards engineering such as the Centre de Recherche en Calcul Appliqué (CERCA), just to name one. The reinforcement of this linkage has been consolidated by the recent success of an application to the NSERC Network Program.

The Institut des Sciences Mathématiques is a federative institution providing advanced graduate training to students of the greater Montréal area. It provides the framework for sharing the best courses, and, thereby, raising the level of students attending a given course. As a result, it enhances the capability of attracting first rate international students. It is very unlikely that such a remarkable structure would have emerged, if the CRM had not been in existence.

The Fields Institute

In but a few short years –it began in 1992– the Fields Institute has established itself on the world stage. It is mentioned in the same breath as MSRI at Berkeley, IMA at Minneapolis and the Newton Institute at Cambridge (UK). This has been accomplished by attracting world leaders, by offering programs of current interest and by energetically soliciting proposals from researchers both inside and outside Canada. Because there are no permanent members, it can react quickly and flexibly to exciting developments (e.g., Andrew Wiles gave a talk on his Fermat work very soon after the result was announced). There is a mixture of special programs and workshops. A broad range of subjects are supported, from Mathematics, Applied Mathematics, Computing and Statistics in particular. There is involvement of active researchers, students, teachers and people from business and government. The events may take place at the permanent building in Toronto or around the country (e.g., Raoul Bott gave a Fields Institute Distinguished Lecture in Cape Breton).

The proposals funded are selected by a Scientific Advisory Panel. The proposers are encouraged to supplement the funding from outside sources and this seems to be occurring. Fields receives funding from NSERC, the Government of Ontario, member institutions and other donors. A publication series, joint with the American Mathematical Society, also brings in some revenue. The building itself came from the University of Toronto.

The Fields Institute is a major success story for science in Canada. Its funding should be stable and settled as far in advance as possible so that there can be an orderly development of programs.

Collaboration between the two established institutes

Collaboration between CRM and the Fields Institute began at the time when NSERC sought a proper mechanism to ensure that both would cooperate on questions of national interest. It was for this purpose that a liaison committee was created, with the result that quite a large number of the proposals of national interest are now jointly funded by the two institutes.

Since NSERC dropped the Conference Grants Program, the Fields Institute and the CRM have become the major selectors and funders of meetings. The broad implications of this concentration need to be monitored, in order to make sure that it does not lead to fewer local initiatives, and to over-concentrated scientific targets.

Outreach and Industrial Partnership

Mathematics is simultaneously a thriving discipline in itself, a foundation of Science, Engineering, and Technology, and a regimen for logical, precise thinking and analysis. Because of the last two characteristics, research in Mathematics has important effects, direct and indirect, on institutions and persons outside the mathematical community. The indirect relationship tends to happen by osmosis or serendipity ; the direct connections result from purposeful initiatives.

For a variety of reasons, including diminishing resources and a perceived reduction in respect, Canadian mathematicians have focused considerable attention in recent years on the advantages of and techniques for outreach to other disciplines and to the community at large. Motivations for outreach activities include bringing young people into the profession, attracting money and other resources, social responsibility, publicizing ideas, and finding novel and interesting problems to study. Many forms of outreach have been explored by Canadian mathematics to create links to industry (academic courses and consulting), government (consulting and training), schools (competitions, workshops), other university departments (interdisciplinary programs, service teaching) and society generally (popular writing). The structures for these outreach programs in Canada include institutes (regional and local), consulting units (applications of mathematics, statistics), societies like the Canadian Applied Mathematical Society (CAMS), the Canadian Mathematical Society (CMS), the Statistical Society of Canada (SSC) and individual enterprise.

The regional institutes are CRM, Fields and PIms. CRM has close ties with other centers whose purposes are technology transfer (in management, finance, and numerical computation) and particular applications of mathematics (risk, decision-making, transportation). Partnerships between industry and CRM support industrial postdocs and fellowships. The Fields Institute has worked principally through conferences and workshops on a variety of subjects —e.g., risk analysis, financial mathematics, control theory, neuroscience, computer graphics— attended by members of government and industry, teachers, and researchers from related areas. PIms, which is just beginning its life as an institute, has already identified and interacted with the private, public, and educational sectors.

In addition, mathematicians at all the universities visited by the committee described various forms of informal, local outreach, primarily involving programs for students and interdisciplinary conferences. World Wide Web access is seen by several

Canadian mathematicians as an efficient means of serving the broader community across a wide geographical area. For example, the CMS's World Wide Web server *Camel* brings information about mathematics into households across the country.

Our committee believes that outreach is a win-win activity for mathematicians and their institutions. Mathematicians can offer powerful concepts and techniques as part of their responsibility for technology transfer, and can also learn about new problems that may well lead to new mathematics. Non-mathematicians can in many instances gain understanding —in the best case, the solution— of their problems.

New Projects

The Pacific Institute for Mathematical Sciences

The Pacific Institute for Mathematical Sciences (PIms) is a cooperative project involving five funding institutions (Simon Fraser University, the University of Alberta, the University of British Columbia, the University of Calgary, and the University of Victoria), all from British Columbia or from Alberta, and a few affiliated institutions. It has mobilized a wide spectrum of mathematicians and scientists in the western provinces in an effort to develop collaborative programs without thinking of having to establish a physical center. Its main aim is to improve the circulation of mathematical ideas inside and outside the academic community, with particular emphasis on the needs of the industrial sector. Creating new partnerships seems its main motto, and it is a fact that some of the universities that participate have already a long record of such outside involvements.

It must be said that, at first, some panel members had strong reservations about any project which would result in the establishment of a new institute. Nonetheless, even these skeptics were won over by the combination of quality, innovativeness, and breadth of support generated by the project. We will come back later to the question of interaction of this new structure with the existing institutes in the next part, when we turn ourselves deliberately towards the future.

The National Network for Collaboration in the Mathematical Sciences

Although this global application to the NSERC Network Program has not been granted support, it is exemplary of the actual working of the Canadian mathematical community. Mathematics is one of the few sciences which has not yet exploded into many subdisciplines. We have already given some arguments to explain this fact (which of course is not at all to be confused with the idea that the discipline could be narrow enough to allow a good mind to embrace the field as a whole ; this was probably the case for exceptional minds in the last century but is no longer conceivable today). Nevertheless, it is a distinctive feature of mathematicians in Canada that they have developed a striking sense of common destiny, and this does represent an exceptional chance for the Canadian society. The whole purpose of the application was to enhance the capability of mathematicians in Canada of interacting between themselves, and also with the outside world, in an effort to diffuse mathematical knowledge and know-how as efficiently and as unobtrusively as possible. The fact that the unique dimension of such a project did not jibe exactly with the rules governing the application scheme should not be a reason for dismissing this remarkable opportunity.

Funding structure

Mathematical research in Canada is getting support from different sources, local, provincial and federal. It is therefore no surprise that the funding situation varies very much from one place to another. The role of the national funding agency, NSERC, is of course very important since it provides a reference. It is running a number of programs from which mathematicians draw resources. We analyze the support given to mathematicians on the basis of the mechanism used to get the funds. We begin with the most important one, namely the NSERC individual research grant allocation.

NSERC individual research grants

The individual research grants program is the mainstay of NSERC's mathematical research funding. Proposals for this program are first subjected to peer review, both inside and outside Canada. The comparative judgements involved in their final disposition are then made by a Grant Selection Committee (GSC), a broadly based panel of leaders from the disciplines of the proposals. The engagement of diverse expert opinions at both the review and decision stages is perceived as providing funding decisions of high quality, and it has earned this program the respect and approval of the mathematical community in Canada. There are two GSCs devoted to Mathematics, GSCs 336 and 337 ; in addition, research that has mathematical aspects is also funded by the GSCs for a number of other disciplines, including Statistics, Physics, Computer Science, and Industrial Engineering.

Within a discipline, the fraction of proposals funded and the distribution of funds among them are under the control of the GSC. (This control is subject to constraints concerning, for example, the funding of proposals by new researchers; these constraints, however, do not appear to be controversial or unduly restrictive.) This fraction and distribution appear to be roughly similar in Mathematics and the other disciplines for which data was available. Thus the entire distribution of grant sizes (including unsuccessful proposals) for Mathematics appears to be a scaled-down version of the distribution for other disciplines. This scaling supports the hypothesis that the small average grant size in mathematics is due largely to lower levels at which Mathematics, as a discipline, is funded and is not a consequence of an effort on the part of the Mathematics GSCs to spread the available funding over a larger fraction of proposals.

Over all disciplines, there is a fairly large fraction of successful proposals and a fairly large ratio between the largest and smallest awards to successful proposals. This distribution is partly attributed to the need to support the Canadian practice of providing advanced education (including post-graduate education) at geographically dispersed locations; and the distribution has virtually unanimous support from the research community (even from those who might be expected to benefit in the short term from a shift to a more concentrated funding distribution). One of its obvious advantages is the possibility to give some support to almost every promising young researcher.

The separation of disciplines, with their GSCs having separate pools of funds to administer, provides a mechanism for taking into account the differing needs and values of disciplines. It raises the problems, however, of ensuring equity among disciplines and of fostering interdisciplinary research. The natural state of affairs, and one that has clear advantages for the quality of research, is that researchers submit their proposals to the GSCs covering the discipline that they feel is best prepared to appreciate and evaluate their work. This practice derives from the very fact of distinguishing disciplines within the broader enterprises of Natural Science and Engineering. Two phenomena arise from it, however, that deserve further attention. First, if there are inequities in the funding of the various disciplines, these may introduce a distorting pressure on patterns of submission, with researchers submitting to richer GSCs proposals that they might actually think more appropriate for poorer ones. Second, if the funds at the disposal of a GSC are fixed, independently of the portfolio of proposals it adjudicates, then it has little incentive to undertake any but the most obviously outstanding of interdisciplinary proposals.

One further issue that should be emphasized is the importance of allowing the utmost flexibility in the use of individual research grant funds. It is this flexibility that allows diverse disciplines to tailor the research grant program (which must of course be structured in a discipline-neutral way) to best serve the needs of the disciplines. And it is this flexibility that allows the broadly supported individual grant program to achieve goals that would otherwise require a plethora of administratively complicated special programs. At a time when sources of funds are under increasing pressure, flexibility can maximize the benefits accruing from funds by putting spending decisions in the hands of those who have been judged most worthy by the community.

The committee was struck by the fact that almost every mathematician who could seek support outside GSCs 336/337 would do so. The systematic answer that the committee got was that this could bring him or her a grant of a more substantial size. There can be various explanations, and applied mathematicians have stressed that, as part of the Mathematics community, they are concerned that the often substantial needs for computational and human resources to conduct research with applications in mind are not well enough recognized. Nevertheless, recognizing that many leading applied mathematicians are funded by NSERC outside the committee officially labeled as "Mathematics" is very important, since, unquestionably, these (often outstanding) researchers form a significant part of the Canadian mathematical scene. It is fair to say that the support of mathematicians engaged in subjects at the boundary between Probability and Statistics is split between the Mathematics and Statistics grant committees without causing any difficulties.

Other NSERC programs

Mathematicians are also getting support to buy equipment. As we explained earlier, this is no longer a side consideration of the development of mathematical research, in particular for mathematicians involved in applications. For this too, it seems that the lesser share going to mathematical applications encourages similar behavior to that previously mentioned, namely, to incite mathematicians, as often as they can, to apply in conjunction with projects in other disciplines.

A program which was of considerable importance to mathematicians and that NSERC canceled as part of a process of simplifying its procedures was that supporting attendance at conferences and workshops. Mathematicians were receiving a fairly large share of it, and its cancellation has had a significant adverse impact on the travel possibilities given to scientists from the more remote areas. This should be taken into account when discussing the support given globally to the discipline.

Provincial support

Provinces are not behaving in a uniform way as far as the support of research is concerned. Their attitude may also vary in time since they are not all facing the same budget constraints at a given moment. Support that mathematicians receive from their provinces varies in level and form. It is very substantial in Québec for example, Ontario engaged itself in the support of the Fields Institute, and the endorsement by Alberta and British Columbia officials of the PIms initiative is also a strong sign. Attitudes towards supporting foreign graduate students also vary a lot, again as a function of place and time. The new restrictive stand seems to handicap considerably the ability of some departments to attract good students.

Other types of funding

It was very difficult for the panel to evaluate the impact of private funding, coming either from companies, through research contracts, or from foundations. It is nevertheless clear that such support plays an important role in some exemplary cases (e.g., the support given by Mr. Aisenstadt to the CRM both to run some specific programs and to build and equip its new headquarters).

Positions

As explained before, in spite of the fact that mathematicians have increased needs for some basic equipment, their main resources are human. As a result, for any mathematical community, the possibility of inviting or hiring colleagues, junior or senior, is fundamental. Without a fairly steady flow of positions, long term or even short term, the mathematical entreprise cannot thrive. A shortage of positions can have several effects :

* The most immediate one can be an increase in the teaching load (which, when compared with other sciences, is most of the time higher for mathematicians engaged in research). There exist for other sciences agencies whose mission is solely devoted to research.
* The diversion from mathematical sciences of the most talented students. Even when this diversion occurs relatively late, it usually takes place before the individual has ever engaged in actual research.
* Last, but not least, it can deprive the field of the dynamism and energy that young new Ph.Ds can provide. The young are indeed in the best position to innovate, and they bring, almost effortlessly, a high level of creativity and a mix of ideas on which renewed curricula can be based.

Towards the Future

An overall perspective on the situation of Canadian Mathematics must include an evaluation of its ability to react to future challenges. This is of course a very difficult exercise in that it forces the Committee to express views about the evolution of Mathematics, possible new roles for this discipline, and how to cope with demographic problems and stimulate the job market.

Since none of these issues is completely in the hands of mathematicians, a major future challenge is to make their case to politicians, administrators, scientists from other disciplines, and, more broadly, the general public. Such an investment in increasing awareness has already begun in Canada, in particular in campaigns launched by the existing institutes and other projects in preparation. Such activities probably need to be conceived on a larger scale, but at least Canadian mathematicians understand the need for better communication about their work.

The Evolution of Mathematics

It is impossible to separate predictions about the future of Mathematics from more general predictions about the future of our societies. So far, Mathematics and its progeny, like Logic and Statistics, have provided effective tools for weighing the reliability of our observations about the past and assessing the validity of our assumptions about the future. Many life-and-death decisions in the modern world are in fact based on confidence in mathematical models.

The contributions of mathematical research to society take varied forms, explicit and implicit, direct and indirect. Many of the most important benefits were completely unanticipated when the fundamental research was carried out; for example, the mathematics of the CAT scan arises from abstract questions about how many two-dimensional cross sections are needed for reliable reconstruction of a three-dimensional image. This element of serendipity may be discomfiting to those who like detailed planning and tidy cost/benefit ratios, but it is simply unavoidable.

If forced to hazard a guess about which Mathematics being done today will be said, in the future, to have been “prescient”, one would want to hedge one’s bet by placing it on a mixture of the “traditional” and “non-traditional”. In the former category, one would be foolish not to assume that Algebra, Number Theory, Geometry, and Partial Differential Equations will continue to be the source of powerful and surprising insights and applications. In the latter category, probably the strongest candidates come out of the analysis of dynamical systems (itself, one of the most traditional and venerable branches of Mathematics) in one of its many disguises. An example of such a field is Optimization. The contributions of mathematicians to this field appear inexhaustible, and they seem to be recognized and put into use almost immediately. This research also catches the attention of mathematicians, in part because even the most abstract structures seem to allow the formulation of questions of optimality.

New Frontiers for Mathematics

The first frontier of Mathematics is Mathematics itself. As we have already said, the intimacy that various parts of the mathematical entreprise enjoy is sometimes so close and so unexpected that all of a sudden completely new perspectives appear where, before, only technical problems seemed at stake. Just to give an example of such new proximities, let us mention that Statistics and Geometry have gained in recent years some common objects to study, a feature which was not at all expected. This requires that enough students get a broad training, and that the evaluation of departments appreciate efforts to diversify the population of researchers and teachers present, and encourage such attitudes.

In the foreseeable future, the combination of new mathematical algorithms and ever-increasing computing power is almost certain to bring extraordinary gains, as well as entirely new capabilities, to critical real-world problems. The following examples are meant to illustrate new and evolving roles for Mathematics in specific applications.

Medicine

Advances in image processing, such as improved techniques for discriminating between noise and small but real perturbations in the data, depend on research in mathematical algorithms. Human judgment plays a crucial role in many image-based decisions, so that major gains in speed as well as reliability are important.

In medicine, for example, CAT and PET scans are well known diagnostic tools (based on Mathematics) that detect malignancies and structural anomalies. However, the time required to obtain them has limited their applicability to offline situations—typically, the physician does not see a scan result until several hours after the scan occurs. Recent progress in mathematical techniques for analyzing ill-conditioned noisy data, implemented on high-performance computers, have allowed brain scans to be viewed and adaptively controlled during surgery. For the first time, surgeons can obtain details about "interesting" regions based on their observations of brain tissue while the patient is under anesthesia.

This example is only one of many opportunities for Mathematics to address medical applications. The areas of medicine and biology are especially promising for Canadian mathematicians, who have already developed connections with physiology and mathematical biology —working on, for example, dynamical systems, pattern identification, and nonlinear PDEs.

Materials

Because prototypes are extremely expensive to produce, mathematical simulation has become standard practice in materials science and engineering —for instance, in predicting material properties, damage, and degradation, and in nondestructive testing. Without new Mathematics, there is no hope of meaningful progress on problems that involve enormously varying scales, from far-field geometry to subwavelengths ; raw computing power is insufficient (by orders of magnitude) to address the size and complexity of key interactions that occur in nonasymptotic regimes where classical formulas fail. Mathematicians across Canada have well-developed expertise in a variety of mathematical issues in materials science —for example, modeling crystal growth, propagation of interfaces, and singular perturbations— and actively collaborate with materials scientists.

Transportation and scheduling

Operations research (the mathematical science of decision-making) and the closely related field of industrial engineering include work on production, scheduling, inventory, and risk analysis (among other areas). For many industries and government entities today, competitive pressures are forcing increased attention to gains in efficiency ; in some instances, a seemingly small percentage increase in revenue or decrease in cost can determine profitability or even survival. As a result, interest will continue to grow throughout the world in associated mathematical techniques, such as modeling and formulation, discrete and continuous optimization, and equilibrium. The Canadian Mathematics community is well connected with these activities in both academic and industrial settings.

The Challenge of Demography

Most Canadian universities are suffering under recent, current, or impending budget cuts as a result of declining enrollments, new emphasis on regional and community colleges, and Provincial and Federal efforts to balance their budgets. Mathematics departments do not appear to have been affected more adversely than other science departments, and the Department chairs have been making thoughtful decisions as to how to proceed during a period of retrenchment. Although there is no immediate cause for alarm, some of the trends are worrisome. Department chairs should be encouraged to seek budgetary solutions which do not involve the use of adjuncts to teach service courses : in the United States, this practice threatens to lead to an unhealthy decrease in the number of academic positions available in Mathematics.

Despite the well publicized shortage of academic positions in Mathematics, the graduate students with whom we spoke were almost unanimous in their enthusiasm for their subject and in their optimism for the future. Most of them are aware of the opportunities for mathematicians which lie outside academia, and their expectations appear much more realistic than they were, say, five years ago when the crisis in the academic job market first became apparent.

Discussions within departmental research groups which specialize in applied Mathematics, combinatorics, or computer-related Mathematics indicated that these fields attract many graduate students, and that these students have little difficulty finding jobs in industry and in the financial sector. The situation in pure Mathematics is more complex since graduate students in these fields have few non-academic options (unless they spend a year or two to “retrain”, in which case they appear to do very well). Nevertheless, much of the highest quality Canadian mathematical research takes place in these pure areas : academic positions are difficult to find and the major departments are hiring only at the highest level. Several leading research mathematicians indicated to us that they had a difficult time attracting quality students. Moreover, the vast majority of the top Canadian mathematical researchers received all or a significant fraction of their graduate education abroad (usually in the U.S.). We do not view this as unhealthy : only the very best students should be encouraged to pursue a career in research Mathematics.

Some smaller departments run Ph.D. programs which turn out scholars who hope to find academic jobs but who are poorly equipped to compete in the current job market. NSERC's emphasis on "training of highly qualified personnel" probably does not help this situation because it encourages faculty (who may themselves be world-class researchers) at smaller universities to supervise such students.

Job Markets and Adaptation of Training Programs

For the past several years, the employment prospects for new mathematicians have been difficult throughout the U.S. and Canada. With the downsizing that has taken place in Canadian Universities and is forecast to continue, traditional academic positions in Canada are few in number, and are very difficult to obtain. New mathematicians must look for employment opportunities outside of academia – in locations such as industrial laboratories ; the financial, banking, and insurance industries; computer and software companies. Canadian Mathematics departments have taken steps, and plan more steps, to help their students begin this transition as a part of their graduate education. Many departments now require that all students at both the Masters and Ph.D. levels obtain significant computer skills as a part of their graduate education. Departments are also attempting to improve the communication skills of junior mathematicians. Several departments are beginning new Masters level programs in financial Mathematics. Useful contacts are being established between the Mathematics departments and parts of the banking and finance industry – an industry that has employed many mathematicians at all levels in recent years. The Institutes have initiated programs designed to enhance contact and collaboration with industry. For example, the Fields Institute has joint programs with the financial industry in the Toronto area. Many of the plans of PIms and the National Network focus upon industrial outreach. It is the impression of our committee that Canadian Universities are somewhat ahead of their sister institutions in many countries, notably the U.S., in some of these activities.

One feature worried members of our committee : As in the U.S., there are groups of junior mathematicians holding post-doctoral positions for several years, in the hope of eventually obtaining an academic position. Members of the committee wondered, particularly in cases at the regional Universities, if these hopes were realistic. If not, our junior colleagues should not be encouraged to remain in these temporary post-doctoral positions, as precious years are in danger of being wasted.

Evaluating Mathematics

The panel discussed possible "numerical indicators" for "objective" assessment of quality. Many funding organizations are turning to the Science Citation Index (compiled by the Institute for Scientific Information) in order to evaluate the progress of a given discipline.

The Citation Index is particularly inappropriate as a vehicle for evaluating progress in the mathematical sciences for several reasons. The Institute for Scientific Information has determined that mathematical journal articles have a half life greater than 10 years, and many suspect that a more accurate number is 20 years. (The half life of an article is the number of years one must go back in order to account for half the total citations to the given article.) In spite of this finding, it is a fact that the Citation Index only publishes citation information for the last 5 years. This gives rise to enormous errors and fluctuations in the computation of total citations.

Mathematicians have a very different attitude towards citations than other scientists. A typical paper in Biology (for example) has almost 10 times as many citations, per page of text, as a typical paper in Mathematics. Moreover, citations in Mathematical papers are not primarily used to attribute credit or to survey the current literature, but rather to refer the reader to further information (documented proofs, algorithms, unifying interpretations, expositions) which may be needed in order to understand the current paper.

When I.S.I. was asked to rank 500 Mathematics Departments throughout the world, according to total citations, strange and somewhat random results were obtained. This committee agrees with the conclusion reached by the United States National Academy of Sciences in their National Research Council report, "Quantitative Assessments of the Physical and Mathematical Sciences" (1994) in which it is stated that "...the Commission's initial optimism about the benefits of greater reliance on or systematic use of quantitative measures in the assessment of scientific disciplines was unfounded."

These facts emphasize the absolute need of being able to rely on the peer review system, and of avoiding its short-comings. This requires in particular taking great care to bring in enough outside views to avoid any risk of transforming the research enterprise into a routine and self-serving activity.

The Need for Appropriate Schemes for Financing

Some of the recent problems between NSERC and the Canadian mathematical community seem to have arisen from misunderstanding that can and should be corrected. Although the amounts of money to be mobilized for the support of Mathematics are substantially smaller than the ones required by experimental disciplines, some of the characteristic features of mathematical research, perhaps less significant for other sciences, should be emphasized. It may be appropriate here to name two :

* the role of personal exchanges, which are also needed in other fields but play a particularly significant role in Mathematics because of the special nature of mathematical production ;
* the role of documentation, because of the need to rely on formal proofs.

Another issue—funding outside the Mathematics GSCs—highlights the contradiction faced by Canadian mathematicians involved in cooperative projects, and is related to a more general problem connected to the evaluation of multi-disciplinary projects. The systematic way in which mathematicians feel compelled to apply to non-mathematical GSCs as soon as they do some interdisciplinary work proves that the basic level of funding for Mathematics projects is lower. Indeed, this disincentive may result in proposals being assigned administratively to GSCs in ways that satisfy neither the proposers nor the committees. These problems cannot be addressed by redefining disciplinary boundaries or by administratively redistributing proposals among disciplines; these attempts would only undermine the sense of community which each discipline has among its members, and which is of enormous importance to their prosperity.

Rather, these problems must be solved by ensuring equity in funding among disciplines, and by establishing mechanisms for the appraisal and funding of interdisciplinary research. Proposals that are frankly interdisciplinary (that is, those which clearly cannot be evaluated responsibly within the framework of any one of the recognized disciplines) might be handled by a special Interdisciplinary Committee. The number of such proposals should be small enough for them to be given the attention they require. This mechanism does nothing, however, for modestly interdisciplinary proposals, for which there may be one obviously appropriate disciplinary home, but for which it is important to take into account the values and judgements of one or more other disciplines in order to reach an appropriate decision. The phenomenon of interdisciplinary research will be of growing importance in the future, as disciplines strive to best take advantage of each other's progress, and it is essential that research institutions and funding agencies develop mechanisms to encourage this research and to provide proper evaluation for its support.

Another issue has to do with the dynamic unity of mathematics, and its great importance in the future development of the discipline. In this respect, the mathematics institutes play a very important role, and the proper mechanism should be found to finance such structures. The procedure should make it possible to plan sufficiently in advance for proposals coming from the community to be able to mature. The Committee feels that the programs of the two existing institutes, CRM and the Fields Institute, and the newly founded PIms, need to be carefully coordinated in order to address the needs of the whole community. In view of the very special geographical and political structure of Canada, it appears to the Committee that the existence of PIms is an essential ingredient in stabilizing the Canadian mathematical institute structure, in particular because of the perspectives set out in the programs that it has started to run. The mechanism to be put in place for future financing should explicitly encourage cooperation between these structures.

In view of the possible shortage of Ph.Ds that could follow a long period of tight job markets, sufficient support for the training of Ph.Ds is required. We also note that the need to attract talented students may justify earlier support. In particular, programs allowing summer fellowships for undergraduates involved in preliminary research work seem to have been very successful. They also give to the most dynamic departments an opportunity to develop very personalized scientific work, something which is of great value, even when conducted on a small scale. The amount of money required to run such programs is not very large, and their potential impact sets them high on the list of activities for which the appropriate funding mechanism remains to be found.

Annex : Terms of Reference

Objectives

To provide the mathematics community and decision makers with an up-to-date "health of the discipline" statement and, in accordance with NSERC's mandate, develop a vision for the future of mathematical research in Canada, with an eye to the impact of this research on education and on technology.

Mandate

To review the health and progress of Canadian mathematical research both in the context of international mathematical research and in the context of Canadian science.

To recommend any measures, such as the structuring of research funding, that it deems important for maintaining and strengthening the vitality of Canadian mathematics.

Terms of Reference

To :

- *assess the health and progress of the discipline in Canada*
- *assess the impact of the discipline*
- *assess the institutional structures of the discipline in Canada*
- *assess demographic and geographic characteristics that affect the development of the discipline in Canada*
- *evaluate the funding structure of the discipline and funding strategy of NSERC's Grant Selection Committees*
- *suggest appropriate ways, such as indicators, of evaluating the discipline's progress and impact*
- *assess the discipline's current communication and collaboration strategy, in its relation to cognate disciplines and to the scientific community at large.*

Roles and Responsibilities

The MLC and NSERC staff will develop the mandate and terms of reference for the review committee. The MLC and NSERC staff will also jointly identify members for the review committee and establish a timetable for its activities. Membership of the review committee (chair, members, convenor) is to be agreed upon by NSERC and the MLC, before potential members are contacted. The Review Committee will have an opportunity to suggest changes to their mandate before it is finalized. No specific role is planned for the MLC while the review is underway. NSERC will be expected to provide, when available and appropriate, information on mathematics funding, program structures and policies, and budget allocations. The review committee should consist of :

A Chair

A non-Canadian mathematician familiar with the Canadian scene. His/her responsibilities will include :

- *chairing the review committee work, ensuring the orderly and complete evaluation of the material submitted to the committee and the transmission of a final report to NSERC and the mathematics community. The process includes ensuring that all important aspects of the review are considered and that a committee consensus is reached.*
- *organizing (or overseeing delegated organization of) the review committee meetings and other activities*
- *directing the preparation of the committee's report*
- *acting as spokesperson for the review committee in dealing with NSERC and the MLC.*

Members

Six to eight members (of which 2 or 3 Canadians) comprising 4 or 5 eminent members of the international mathematical community, 2 or 3 "users and/or collaborators of mathematics", coming either from scientific disciplines such as physics, statistics, computer science, engineering or biology, or from industrial backgrounds with a strong mathematical slant. The members will participate in the review committee's meetings and other activities and can be called upon to prepare their specific part of the report.

A Scientific Convenor

Ideally a senior and respected member of the Canadian mathematical community, whose task will be to :

* *request and gather materials (discipline self-study and others) for the review committee*
* *provide the committee with knowledge and information about Canadian research in mathematics and its context*
* *assist the chair in preparing for and organizing the meetings and other committee activities*
* *arrange for detailed notes to be taken on the deliberations*
* *gather and assemble individual sections of the draft and final reports for approval by the review committee. The convenor would be assigned the necessary secretarial assistance at his/her home university.*

The Self-Study

At this preliminary stage it is proposed that the discipline self-study be composed of the following sections:

* *Mathematical research in Canada: a document along the lines of the Pure and Applied Mathematics GSC Allocation Report (by area, by department, a list of notable Canadian achievements), incorporating the result of departmental submissions*
* *Mathematics education: undergraduate, graduate and post-graduate. Care should be taken not to exceed the mandate which has to be focused on research.*
* *Structures: the Canadian mathematical community and its organization: departments, CMS, CAMS. The research centres and their role*
* *The financing of science in Canada and the financing of mathematics*
* *Mathematics and its interaction with other disciplines and with industry*
* *Planning the future: directions, goals and what our discipline needs to reach them.*

The body of the document should be reasonably concise. Appendices would include, for example, the statistical information compiled by NSERC for the re-allocation, the tabulation of NSERC grantees in mathematics, and other information judged pertinent. The preparation of the self-study would be supervised by the scientific convenor, with significant input from the mathematics community and from NSERC. The self-study would naturally address the Committee's terms of reference.

Tentative schedule

Activity	**Timetable**	**Comments**
Membership selection	*January-March 1996*	*Once completed, briefing of Chair*
Review Committee has a teleconference or e-mail exchange	*April 1996*	*Discuss mandate/terms of reference, process, sources of information, RFIs, deliverables*
Material gathering	*May-August 1996*	*Convenor sending RFIs ; organizing and assembling submissions*
Mailing of advance material to committee members	*early September 1996*	*Convenor sending material to members*
Members reviewing material	*September 1996*	
First meeting of the Review committee	*October 1996*	*Discussion of material ; assess need for more information ; assignment of responsibilities and deliverables (i.e. draft report) for next meeting*
Second meeting of the Review committee	*December 1996 (7-9 in London, Ont.)*	*Review of draft report ; assignment of responsibilities and deliverables for final report*
Submission of final report	*February 1996*	*Final editing ; printing ; submission*

Structure and Content of the Review Committee Report

The structure of the report should reflect the terms of reference and include an analysis of each item, conclusions and recommendations for each item.

* *Health and progress of the discipline*
* *Impact of the discipline*
* *Institutional structures*
* *Demographic and geographic characteristics*
* *Funding structure of the discipline and funding strategy of the NSERC Grant Selection Committees*
* *Methods of evaluation, such as indicators*
* *Communication and collaboration strategy.*