

---

## Symmetry in Geometry

### Symétrie en géométrie

(Org: **Ted Bisztriczky** (Calgary), **Ferenc Fodor** (Szeged, Hungary; Calgary), **Richard K. Guy** (Calgary) and/et **Asia Weiss** (York))

---

---

**CATHARINE BAKER**, Mount Allison University, Sackville, NB

*Affine planes, symmetry and 3 – e.c. graphs*

A graph is 3 – e.c. if, for each triple  $S$  of vertices and for each  $T \subseteq S$ , there exists a vertex not in  $S$  which is adjacent to all vertices of  $T$  and to no vertices of  $S \setminus T$ . The structure and symmetry of an affine plane provide us with tools for constructing families of 3 – e.c. graphs on the point set of the plane and for determining when the resulting graphs are non-isomorphic.

This is joint work with Anthony Bonato (Wilfrid Laurier), Julia Brown (York) and Tamás Szőnyi (Eötvös University).

---

**LEAH BERMAN**, Ursinus College, Department of Mathematics and Computer Science, P.O. Box 1000, Collegeville, PA 19426, USA

*Symmetric Configurations*

A geometric configuration  $(p_q, n_k)$  is a collection of  $p$  points and  $q$  straight lines, usually in the Euclidean plane, so that each point lies on  $q$  lines and each line passes through  $k$  points. This talk will discuss configurations with high degrees of symmetry; in other words, configurations in which the number of symmetry classes of points and of lines formed by isometries of the plane mapping the configuration to itself is small. Of particular interest will be *astral configurations*, where a  $(p_q, n_k)$  configuration is astral if it has precisely  $\lfloor \frac{k+1}{2} \rfloor$  symmetry classes of points and  $\lfloor \frac{q+1}{2} \rfloor$  symmetry classes of lines—the fewest possible.

---

**ANDRAS BEZDEK**, Auburn University, Auburn, AL 36849

*On the number of mutually touching cylinders*

The following problem was posed by Littlewood in 1968. What is the maximum number of congruent infinite circular cylinders that can be arranged in  $R^3$  so that any two of them are touching? Is it 7? It was proved by the author in 2005 that this maximum number is at most 24. The talk will also survey models of mutually touching cylinders and explain the connection of this problem to Gardner's (1959) mathematical puzzle concerning cigarettes.

---

**KAROLY BEZDEK**, University of Calgary, 2500 University Drive NW, Calgary, Alberta T2N 1N4

*On the X-ray numbers of 3-dimensional convex bodies*

P. Soltan (1972) introduced the concept of X-ray numbers of convex bodies. According to a conjecture of K. Bezdek and T. Zamfirescu (1991), the X-ray number of any 3-dimensional convex body is at most 6. I give a proof of this conjecture for any convex body with affine symmetry.

---

**ROBERT DAWSON**, Saint Mary's University, Halifax, NS

*Čebyšev sets in hyperspaces over  $R^n$*

A set in a metric space is called a Čebyšev set if it contains a unique “nearest neighbour” to each point of the space. In this paper we generalize this notion, defining a set to be Čebyšev relative to another set if every point in the second set has a

unique “nearest neighbour” in the first. We are interested in Čebyšev sets in some hyperspaces over  $R^n$ , endowed with the Hausdorff metric, mainly the hyperspaces of compact sets, compact convex sets, and strictly convex compact sets.

We present some new classes of Čebyšev and relatively Čebyšev sets in various hyperspaces. In particular, we show that certain nested families of sets are Čebyšev; as these families are characterized purely in terms of containment, without reference to the semi-linear structure of the underlying metric space, their properties differ markedly from those of known Čebyšev sets. (A conjectured link with symmetry did not materialize; thus this paper has become, in this session, something of a *lucus a non lucendo*.)

---

**ANTOINE DEZA**, McMaster University, Hamilton, Ontario, L8S 4K1  
*Colourful Simplicial Depth*

Inspired by Barany’s colourful Carathéodory theorem, we introduce a colourful generalization of Liu’s simplicial depth of a point  $p$  in  $R^d$  relative to a fixed set  $S$  of sample points, *i.e.*, the number of simplices generated by points in  $S$  that contain  $p$ . We prove a parity property and conjecture that the minimum colourful simplicial depth of any core point in any  $d$ -dimensional configuration is  $d^2 + 1$  and that the maximum is  $d^{d+1} + 1$ . We exhibit configurations attaining each of these depths, and apply our results to the problem of bounding monochrome (non-colourful) simplicial depth. Independently found recent quadratic lower bounds by Barany and Matousek and by Stephen and Thomas are also presented.

Joint work with Sui Huang (McMaster), Tamon Stephen (Magdeburg) and Tamas Terlaky (McMaster).

---

**CHRIS FISHER**, University of Regina, Regina, SK S4S 0A2  
*A Computer Drawing of the Complete Pascal Configuration*

A hexagon inscribed in a conic determines 60 Pascal lines, and these generate a remarkable configuration of 146 points and 110 lines. Although the complete Pascal figure has been studied since the middle of the 19th century, it is only with today’s computer graphics that we are easily able to see it. For the computer to do its job a systematic notation is needed; this was provided by J. J. Sylvester (1844), with improvements from Stanley Payne (1973). Norma Fuller and I will soon have a web page ready to display the figure. In my talk I will demonstrate how the viewer will be able to explore the numerous subconfigurations and see how they fit together to form the whole figure.

---

**BRANKO GRÜNBAUM**, University of Washington, Seattle, WA 98195, USA  
*Are all symmetry groups present in the Alhambra, and related questions*

The question which of the seventeen wallpaper groups are represented in the fabled ornamentation of the Alhambra has been raised and discussed quite often, with widely diverging answers. Some of the arguments from these discussions will be presented in detail. This leads to the more general problem about the validity and meaning of the answers to such questions. The second part of the presentation will deal with the approaches to symmetry in ornaments of various cultures that should replace the mechanical counting of the wallpaper groups that occur. A more reasonable investigation would deal with symmetries as they may be considered and understood by the people of the societies in question. Such an approach to the remarkable orderly decorations of ancient Peruvian fabrics will be presented.

---

**HEIKO HARBORTH**, Techn. Univ. Braunschweig, Germany  
*Integral Distances in Geometrical Figures*

Can one find  $n$  points in dimension  $m$  with pairwise distances being integral (rational)? What is the minimum largest distance (diameter) for  $n$  points, for  $n$  points in semi-general position (no  $m + 1$  points in a hyperplane), and for  $n$  points in general

position (no  $m + 1$  points in a hyperplane, no  $m + 2$  points on a hypersphere)? What about a rational box? What about graph drawings in the plane with straight line edges of integral length?

---

**BARRY MONSON**, University of New Brunswick, Fredericton, NB  
*Down with Symmetry!*

... a little. In a natural way, the faces of ranks 1 and 2 in a 4-polytope  $\mathcal{P}$  provide the vertices of a bipartite graph  $\mathcal{G}$ . Recently, Asia Weiss and I have examined this construction when  $\mathcal{P}$  is a finite, abstract regular (or chiral) polytope of Schläfli type  $\{3, q, 3\}$ . If in this case  $\mathcal{P}$  is also *self-dual*, then  $\mathcal{G}$  must be 3-transitive (or 2-transitive). Here I discuss further work with the additional help of Egon Schulte and Tomaž Pisanski. We show that when  $\mathcal{P}$  is *not self-dual*, then  $\mathcal{G}$  is no more symmetric than it has right to be. Indeed,  $\mathcal{G}$  is a trivalent *semisymmetric* graph, so that  $\text{Aut}(\mathcal{G})$  is transitive on edges but not on vertices.

---

**DEBORAH OLIVEROS**, Universidad Nacional Autónoma de México, Circuito Exterior CU., México DF, México  
*Polygons enclosing point sets*

We will discuss some problems of the following type: Let  $R$  a set of red points and  $B$  be a set of black points in the plane all in general position. Find a simple polygon  $P$  with vertex set  $R$  such that the interior of  $P$  contains as many points of  $B$  as possible.

---

**TOMAZ PISANSKI**, University of Ljubljana and University of Primorska, Slovenia  
*Grünbaum incidence calculus*

One of the key problems in the theory of configurations is the question whether a given combinatorial configuration has a geometric realization. In the last decade Branko Grünbaum systematically investigated a more tractable variant of this problem, namely for which types  $(v_r, b_k)$  do there exist geometric configurations. In a series of papers he developed a large number of interesting constructions that produce large configurations from small ones, thereby giving positive answer in majority of cases. In this talk we present some of his methods as a theory under the name of *Grünbaum incidence calculus*.

---

**EGON SCHULTE**, Northeastern University, Department of Mathematics, Boston, MA 02115  
*The Classification of Chiral Polyhedra*

This talk presents the complete enumeration of chiral polyhedra in Euclidean 3-space. Chiral, or irreflexibly regular, polyhedra are nearly regular polyhedra; their geometric symmetry groups have two orbits on the flags (regular polyhedra have just one orbit), such that adjacent flags are in distinct orbits. There are several (very) infinite families of chiral polyhedra, each either with finite skew polygonal faces and vertex-figures or with infinite helical faces and planar vertex-figures. Their geometry and combinatorics are rather complicated.

---

**ARTHUR SHERK**, University of Toronto  
*A Class of Finite Three-Dimensional Metric Spaces*

Let  $K$  be the cubic extension of the finite field  $F = GF(q)$ , where  $Q$  is any prime power. Being a vector space,  $K$  is also an affine 3-space. The mapping from  $K$  to  $F$  known as the Norm imposes a metric on  $K$ . The unit surface  $C$  in  $K$  is the set of all elements of  $K$  with norm 1. Using the symmetries of the unit surface, we cover  $C$  with a map of type 3,6 (triangular faces, six at each vertex). It then follows that  $C$  is a torus. The covering map has considerable symmetry, but is not regular.